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TP B.11
Shallow-angle contact-point mirror kick system

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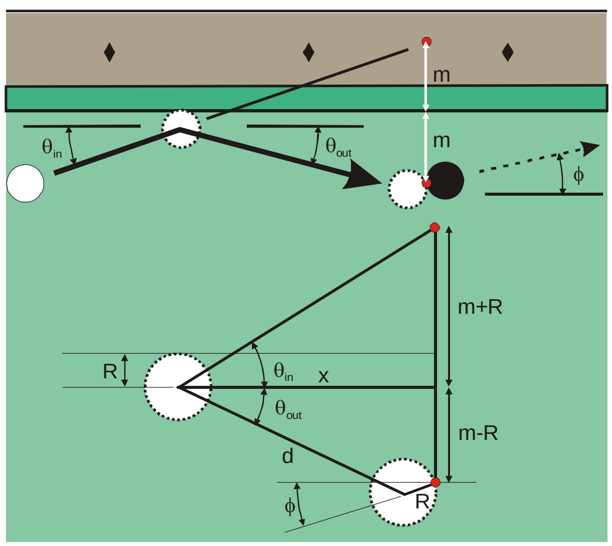
The purpose of this analysis is to compare theoretical and experimental results to show why and when the shallow-angle contact-point mirror kick system is effective. For background information, illustrations, and more information, see my **July '10 BD article**.



$$D := 2.25 \cdot in$$

$$R := \frac{D}{2}$$

From the illustration above,

$$x(\theta_{in}, m, \varphi) := \frac{(m + R)}{\tan(\theta_{in})}$$

and

$$x = d \cdot \cos(\theta_{out}) + R \cdot \cos(\varphi)$$

$$m - R = d \cdot \sin(\theta_{out}) - R \cdot \sin(\varphi)$$

which can be solved (by eliminating d) to give:

$$\theta_{out}(\theta_{in}, m, \varphi) := \operatorname{atan}\left[\frac{m - R \cdot (1 - \sin(\varphi))}{x(\theta_{in}, m, \varphi) - R \cdot \cos(\varphi)}\right]$$

Cushion compression has an effect on the geometry, especially at faster speeds, but I neglect this effects in the analysis. I am assuming the speed is slow enough to not compress the cushion a significant amount. Also, the rebound path of the CB curves after rebound, with this curve being delayed more with faster speed. The effect of curve delay depends on how soon the CB hits the object after rebound. Because of this, I will compare the results to experimental data of both immediate rebound angle off the rail and effective rebound angle, after the CB is done curving.

Here is experimental data for rebound angle vs. incoming angle for a slow rolling CB with no English (taken with Bob Jewett in Fort Collins, March, 2009)

$$\theta e_{in} := \begin{pmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{pmatrix} \qquad \theta e_{out\_immed} := \begin{pmatrix} 5 \\ 10.5 \\ 23 \\ 37 \\ 46 \\ 56 \\ 70 \\ 79.95 \end{pmatrix} \qquad \theta e_{out\_eff} := \begin{pmatrix} 3.5 \\ 8.4 \\ 16.9 \\ 25.2 \\ 35.7 \\ 48.7 \\ 64.1 \\ 75.5 \end{pmatrix}$$

where θe_{in} is the CB angle relative to the rail, θe_{out_immed} is the immediate rebound angle off the rail (measured with the help of a high-speed video camera), and θe_{out_eff} is the effective rebound angle as determined by where the CB hit a rail after rebound.

Here are values for the parameters for the system shots described in my **July '10 BD article** (A: 1-ball shot, 1/2 ball off rail; B: 2-ball shot, full ball off rail; C: 3-ball shot, 1 1/2 ball off rail):

$$\theta_{in} := 5\cdot deg, 6\cdot deg .. 45\cdot deg$$

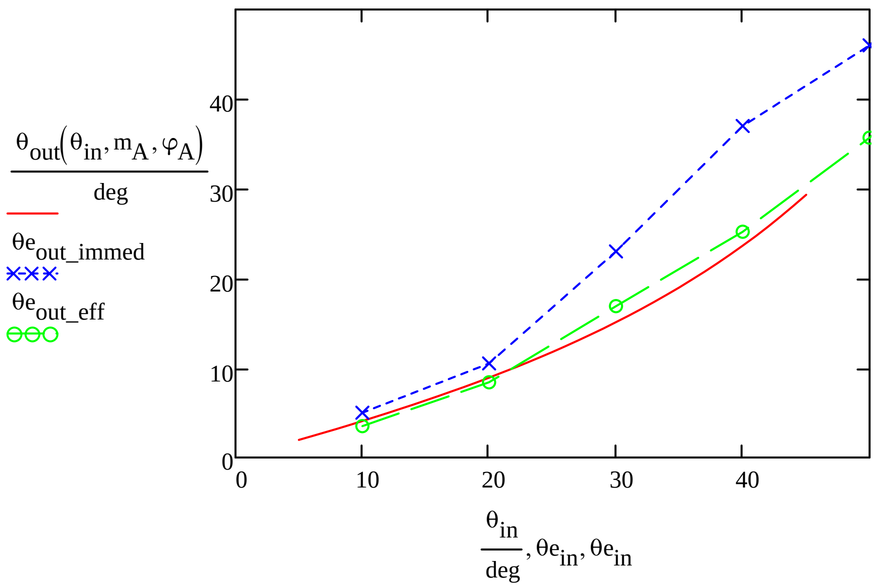
$$\varphi_A := 5\cdot deg \qquad \varphi_B := 7.5\cdot deg \qquad \varphi_C := 10\cdot deg$$

$$m_A := D + R\cdot\sin(\varphi_A) \qquad m_B := (D + R) + R\cdot\sin(\varphi_B) \qquad m_C := 2\cdot D + R\cdot\sin(\varphi_C)$$

When I tested the three shots on my table, shot "A" was over cut for a wide range of angles from 5 to 45 degrees, with the largest over-cut occurring at larger angles. Shot "B" was successful over quite a large range (5 to 37 degrees), with over cut occurring at larger angles. Shot "C" was under cut at smaller angles (5 to 25 degrees), and was over cut at larger angles (35 to 45 degrees). The shot was successful in the mid range (between 25 and 35 degrees), but the speed was critical.

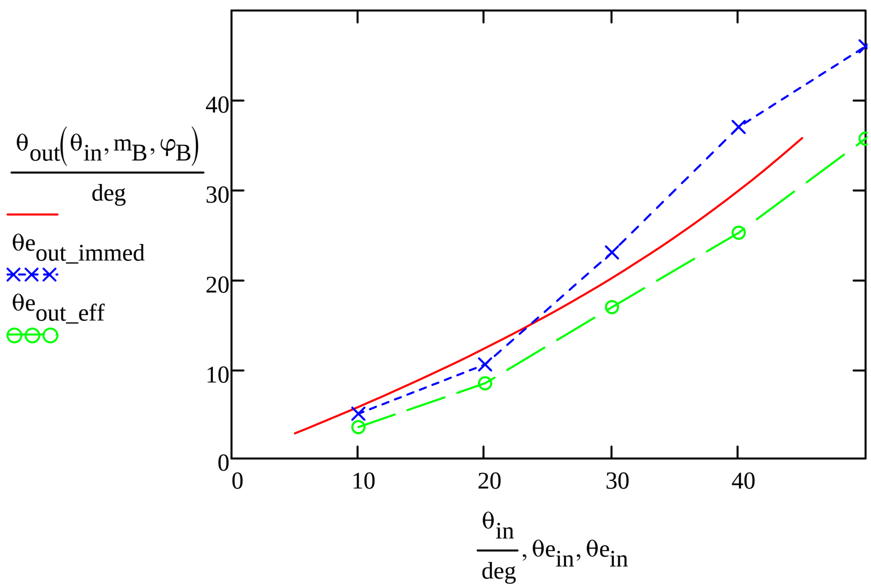
Here are plots comparing the rebound angle predicted by the system to the actually experimental data:

Shot "A" (the 1-ball shot) in the article



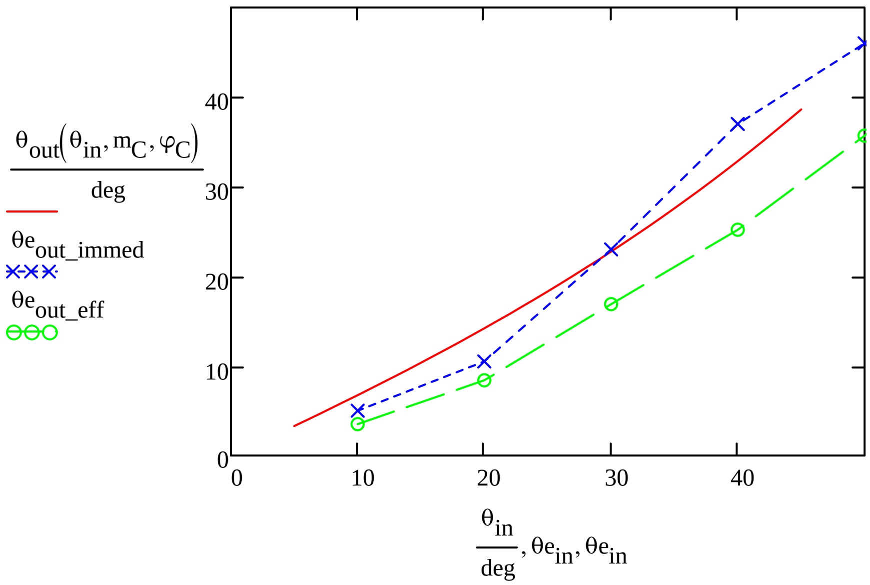
Here, the actual rebound angles (from the experimental data) are larger than the angle predicted by the system, which explains why this shot is typically over cut, and by a wider margin at larger angles.

Shot "B" (the 2-ball shot) in the article



Here, the actual rebound angles (from the experimental data) are closer to and mostly bracket the angle predicted by the system over a wider range, which explains why the system works quite well for this shot.

Shot "C" (the 3-ball shot) in the article



Here, the actual rebound angles (from the experimental data) are smaller than the angle predicted by the system at smaller approach angles, which explains why this shot is typically under cut. At larger approach angles, the immediate rebound angle is larger than predicted, explaining why the shot is over cut at larger angles.