technical proof technical proof

TP A.20
The effect of spin, speed, and cut angle on draw shots

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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Refer to TP A.4 for the background derivations and illustrations.

ball geometry:

$$D := \frac{2.25 \cdot in}{m} \qquad R := \frac{D}{2} \qquad D = 0.057 \qquad R = 0.029$$

coefficient of friction between the cue ball and table cloth:

$$\mu := 0.2$$

gravity

$$g := \frac{g}{\left(\frac{m}{s^2}\right)} \qquad g = 9.807$$

time required for the cue ball to start rolling (cease sliding):

$$\Delta t(v, \omega, \phi) := \frac{2 \cdot v \cdot \sin(\phi)}{7 \cdot \mu \cdot g} \cdot \sqrt{\cos(\phi)^2 + \left(\sin(\phi) + \frac{R \cdot \omega}{v \cdot \sin(\phi)}\right)^2}$$

velocity components when the cue ball starts rolling in a straight line:

$$v_{xf}(v, \omega, \phi) := \frac{5}{7} \cdot v \cdot \sin(\phi) \cdot \cos(\phi)$$

$$v_{yf}(v, \omega, \phi) := \frac{1}{7} \cdot \left(5 \cdot v \cdot \sin(\phi)^2 - 2 \cdot R \cdot \omega\right)$$

the final deflected cue ball angle:

$$\theta_c(v,\omega,\phi) := \operatorname{atan2}\left(5\cdot v\cdot\sin(\phi)^2 - 2\cdot R\cdot\omega, 5\cdot v\cdot\sin(\phi)\cdot\cos(\phi)\right)$$

x position of the cue ball during the curved trajectory:

$$x_c(t,v,\omega,\phi) := v\cdot t\cdot\sin(\phi)\cdot\cos(\phi) - \frac{\mu\cdot g\cdot t^2\cdot\cos(\phi)}{2\cdot\sqrt{\cos(\phi)^2 + \left(\sin(\phi) + \frac{R\cdot\omega}{v\cdot\sin(\phi)}\right)^2}}$$

x position of the cue ball during and after the curved trajectory:

$$x(t,v,\omega,\phi) := \left| \begin{array}{l} \Delta T \leftarrow \Delta t(v,\omega,\phi) \\ x_c(t,v,\omega,\phi) \ \text{ if } t \le \Delta T \\ \left[x_c(\Delta T,v,\omega,\phi) + v_{xf}(v,\omega,\phi)\cdot(t-\Delta T)\right] \ \text{ otherwise} \end{array} \right.$$

y position of the cue ball during the curved trajectory:

$$y_c(t,v,\omega,\phi) := v\cdot t\cdot\sin(\phi)^2 - \left[\frac{\mu\cdot g\cdot t^2\cdot\left(\sin(\phi) + \frac{R\cdot\omega}{v\cdot\sin(\phi)}\right)}{2\cdot\sqrt{\cos(\phi)^2 + \left(\sin(\phi) + \frac{R\cdot\omega}{v\cdot\sin(\phi)}\right)^2}}\right]$$

y position of the cue ball during and after the curved trajectory:

$$y(t,v,\omega,\phi) := \left| \begin{array}{l} \Delta T \leftarrow \Delta t(v,\omega,\phi) \\ y_c(t,v,\omega,\phi) \ \text{ if } t \le \Delta T \\ \left[y_c(\Delta T,v,\omega,\phi) + v_{yf}(v,\omega,\phi)\cdot(t-\Delta T)\right] \ \text{ otherwise} \end{array} \right.$$

Equation for the tangent line:

$$x_{tangent\_line}(t) := \frac{t}{T}\cdot 2$$

$$y_{tangent\_line}(t,\phi) := x_{tangent\_line}(t)\cdot\tan(\phi)$$

Equation for the ball (for scale):

$$x_{ball}(t) := R\cdot\cos\left(\frac{t}{T}\cdot\frac{\pi}{2}\right)$$

$$y_{ball}(t) := R\cdot\sin\left(\frac{t}{T}\cdot\frac{\pi}{2}\right)$$

Plot data:

$\phi\phi := 30\cdot deg$ — cut angle for 1/2-ball hit

$v := 5\cdot\frac{mph}{\frac{m}{s}}$ — average speed in mph converted to m/s

$\omega_{roll} := -\frac{v}{R}$ — average natural roll spin rate

$f\_range := 0, 0.01 .. 1.0$ — ball-hit fraction range

$\phi(f) := asin(1 - f)$

$f(\phi) := 1 - sin(\phi)$ — cut angle and ball-hit fraction relationships (from TP 3.3)

$SRF\_range := 0, 0.05 .. 1.25$ — spin rate factor range (from TP A.12)

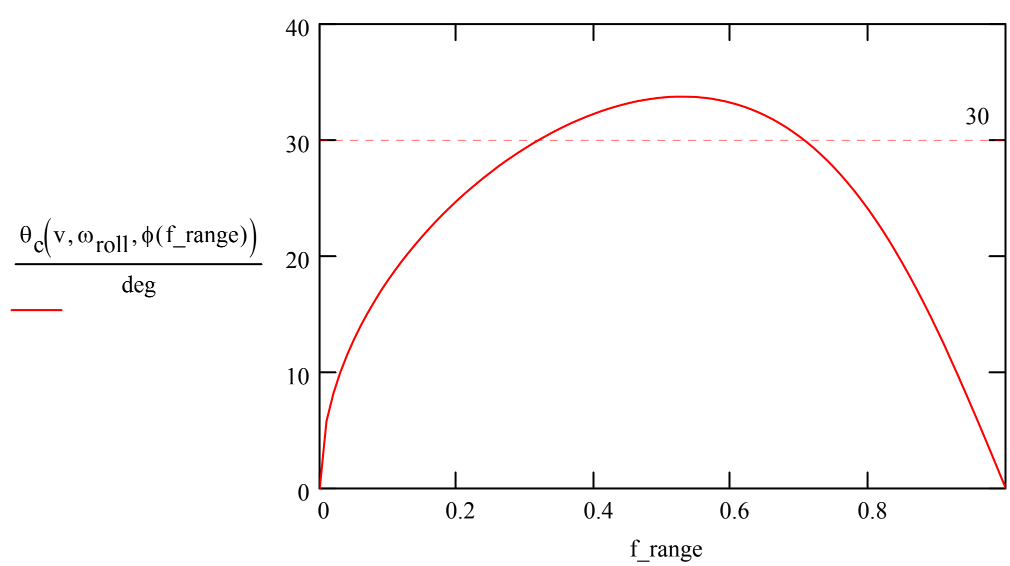
$T := 5$ — number of seconds to display

$t := 0, 0.01 .. T$ — 0.01 second plotting increment

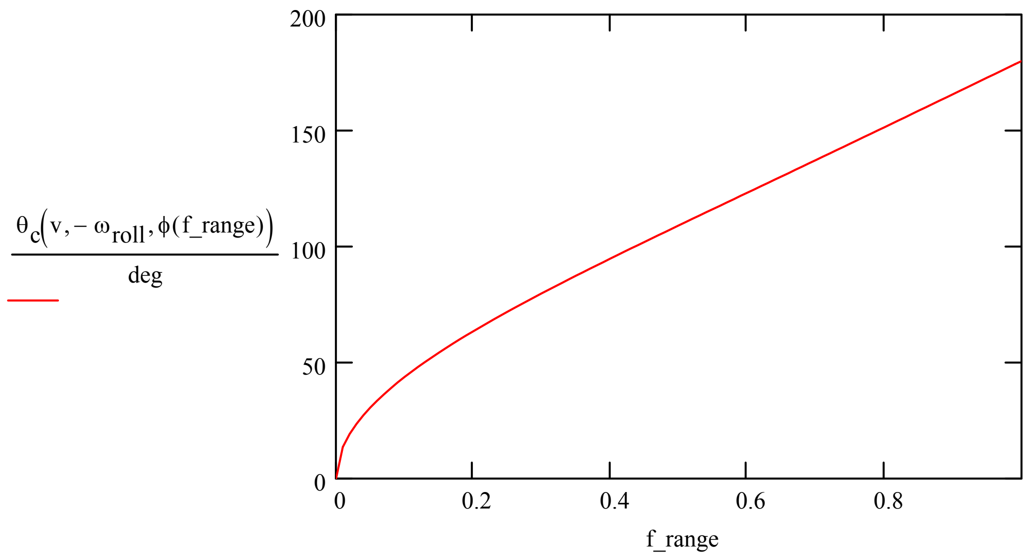
various speeds (in mph, converted to m/s), from slow to very fast:

$v_1 := 2\cdot\frac{mph}{\frac{m}{s}}$ $v_2 := 4\cdot\frac{mph}{\frac{m}{s}}$ $v_3 := 6\cdot\frac{mph}{\frac{m}{s}}$ $v_4 := 8\cdot\frac{mph}{\frac{m}{s}}$

natural roll follow shot (see TP 3.3):



natural (reverse natural roll) draw shot:



For a 1/2-ball hit:

$\phi_{half} := 30 \cdot deg \qquad f(\phi_{half}) = 0.5$

$\theta_c(v, -\omega_{roll}, \phi_{half}) = 109.107\ deg$

Cut angle (and ball-hit fraction) required for the deflected cue ball direction to be perpendicular to the original (aiming line) direction:

$\phi_{90} := 30 \cdot deg$ initial guess (1/2-ball hit)

Given

$\theta_c(v, -\omega_{roll}, \phi_{90}) = 90 \cdot deg$

$\phi_{90} := Find(\phi_{90}) \qquad \phi_{90} = 39.232\ deg \qquad f(\phi_{90}) = 0.368$

Amount of draw (spin rate factor) required for perpendicular deflection at a 1/2-ball hit:

$\phi_{90} := 30 \cdot deg \qquad f(\phi_{90}) = 0.5$

$SRF := 1$ initial guess (natural roll)

Given

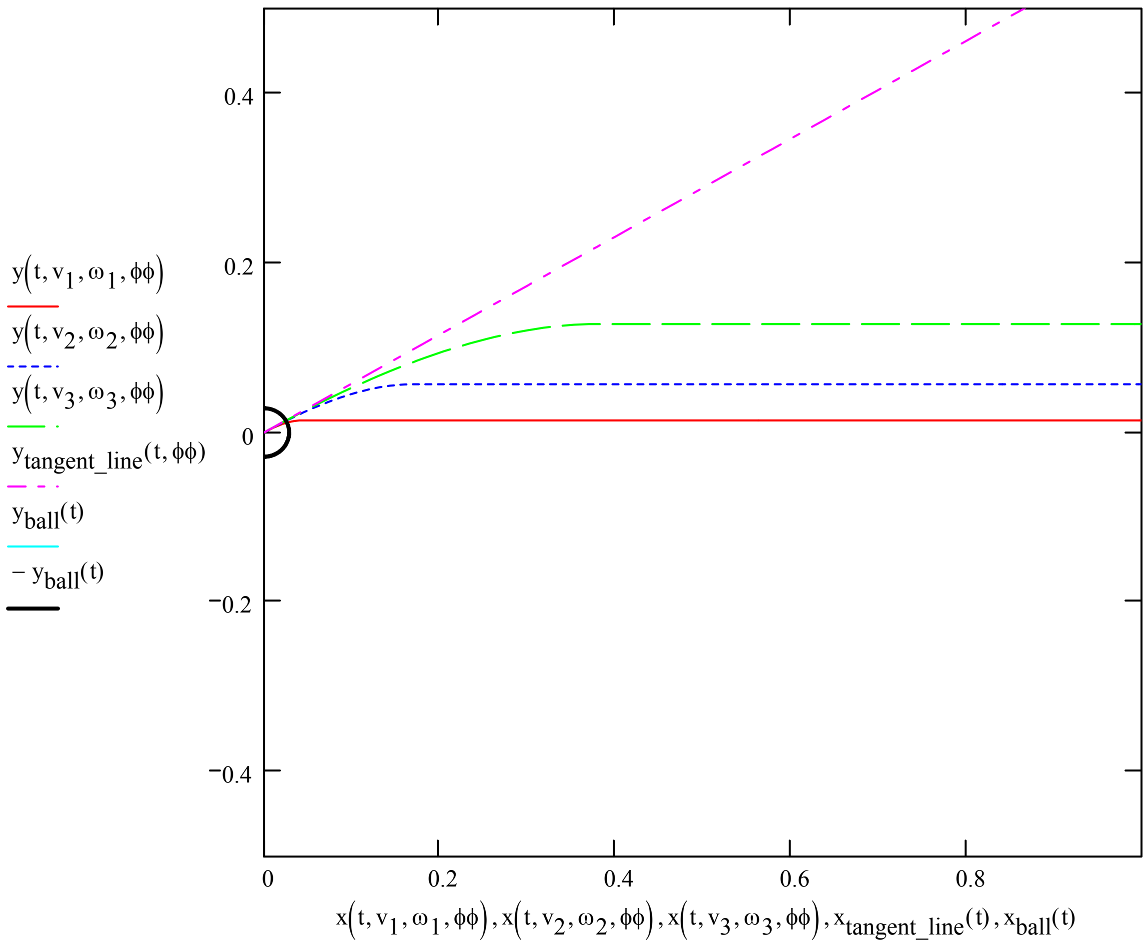
$\theta_c(v, -SRF \cdot \omega_{roll}, \phi_{90}) = 90 \cdot deg$

$SRF\_TYP := Find(SRF) \qquad SRF\_TYP = 0.625$

This will be referred to as a "**typical amount of spin for a draw shot.**" It is 62.5% of the natural roll rate.

various speed 1/2 ball-hit draw shots with typical amounts of spin:

$$\omega_1 := SRF\_TYP \cdot \frac{v_1}{R} \qquad \omega_2 := SRF\_TYP \cdot \frac{v_2}{R} \qquad \omega_3 := SRF\_TYP \cdot \frac{v_3}{R}$$



various ball-hit fraction draw shots with typical amounts of spin:

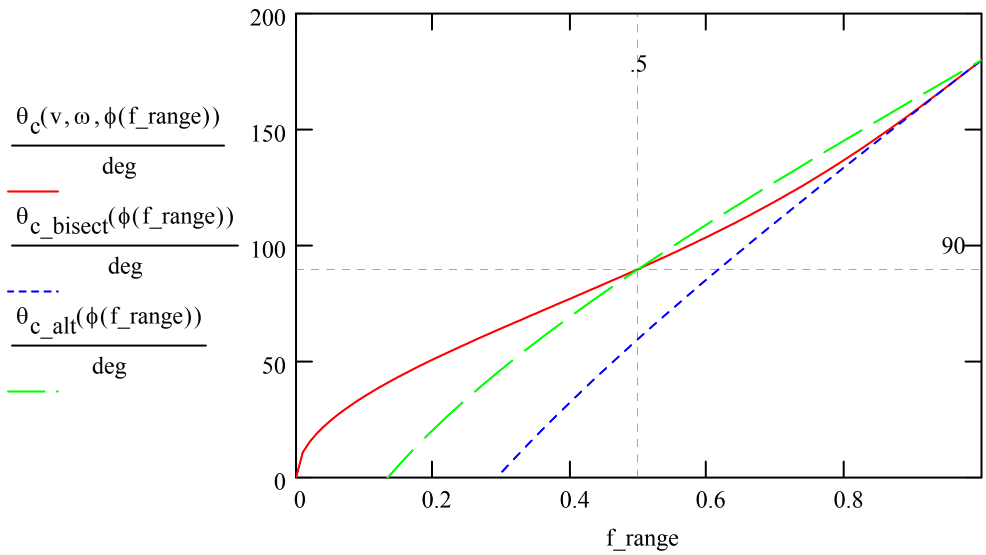
$$\omega := -SRF\_TYP \cdot \omega_{roll}$$

Double bisect draw shot aiming system described in Jewett's October '95 BD article:

$$\theta_{c\_bisect}(\phi) := 180 \cdot deg - 4 \cdot \phi$$

Trisect aiming system proposed here:

$$\theta_{c\_alt}(\phi) := 180 \cdot deg - 3 \cdot \phi$$



1/4-ball hit: $\phi_1 := \phi(.25)$ $\phi_1 = 48.59\ deg$

$\theta_c(v, \omega, \phi_1) = 57.791\ deg$ $\theta_{c\_bisect}(\phi_1) = -14.362\ deg$ $\theta_{c\_alt}(\phi_1) = 34.229\ deg$

1/2-ball hit: $\phi_2 := \phi(.5)$ $\phi_2 = 30\ deg$

$\theta_c(v, \omega, \phi_2) = 90\ deg$ $\theta_{c\_bisect}(\phi_2) = 60\ deg$ $\theta_{c\_alt}(\phi_2) = 90\ deg$

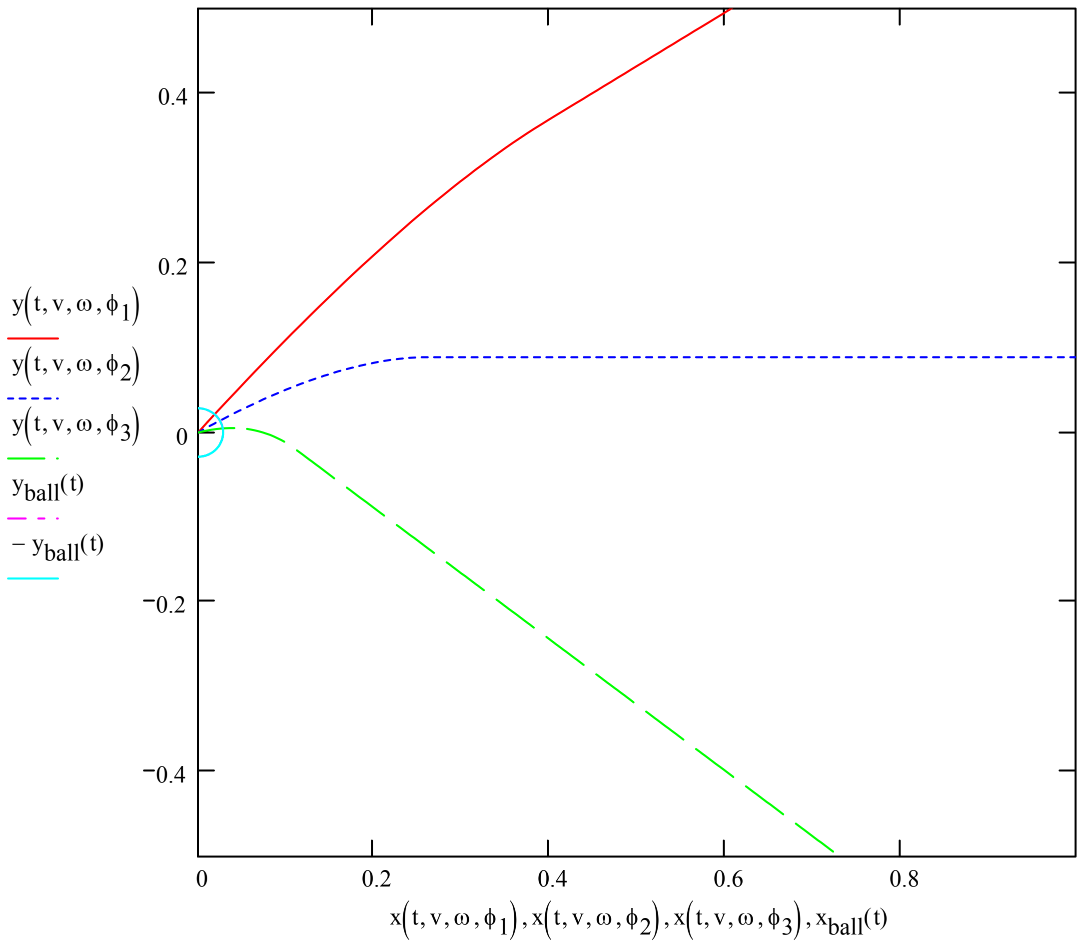
3/4-ball hit: $\phi_3 := \phi(.75)$ $\phi_3 = 14.478\ deg$

$\theta_c(v, \omega, \phi_3) = 127.761\ deg$ $\theta_{c\_bisect}(\phi_3) = 122.09\ deg$ $\theta_{c\_alt}(\phi_3) = 136.567\ deg$

7/8-ball hit: $\phi_4 := \phi\left(\frac{7}{8}\right)$ $\phi_4 = 7.181\ deg$

$\theta_c(v, \omega, \phi_4) = 152.114\ deg$ $\theta_{c\_bisect}(\phi_4) = 151.277\ deg$ $\theta_{c\_alt}(\phi_4) = 158.458\ deg$

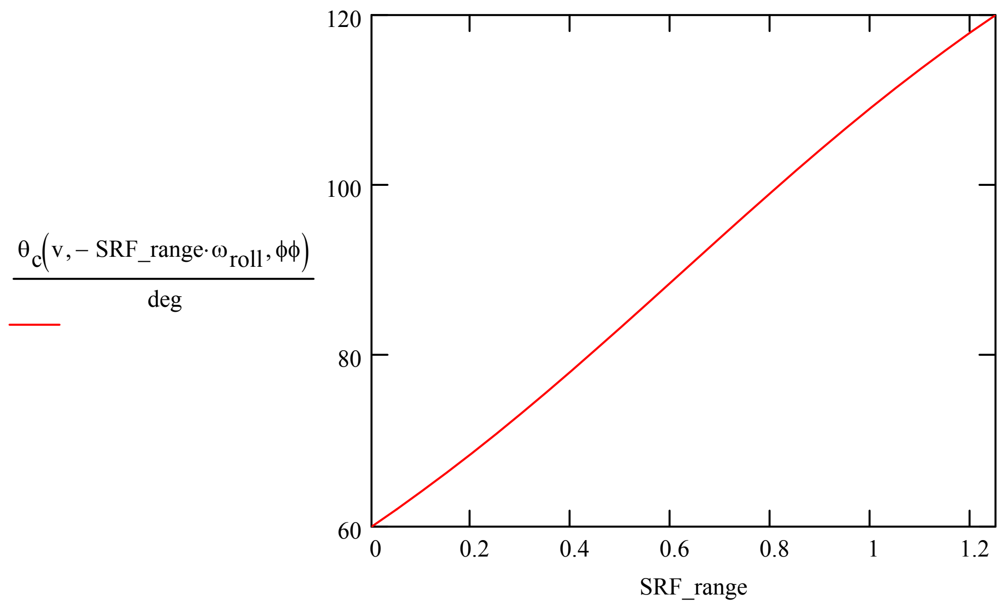
The trisec system works fairly well over a wide range of cut angles (ball-hit fractions). The double bisect system is better for small cut angles (large ball-hit fractions). Both systems are poor for large cut angles (small ball-hit fractions), especially the double bisect method.



0.4
0.2
0
−0.2
−0.4
0
0.2
0.4
0.6
0.8
y(t, v, ω, ϕ1)
y(t, v, ω, ϕ2)
y(t, v, ω, ϕ3)
yball(t)
− yball(t)
x(t, v, ω, ϕ1), x(t, v, ω, ϕ2), x(t, v, ω, ϕ3), xball(t)

half-ball hit draw shots at various spin rates:

$\phi\phi = 30\,deg$ 1/2-ball hit



75% of typical draw shot spin (25% less than typical):

$SRF_1 := 0.75 \cdot SRF\_TYP$ $SRF_1 = 0.469$ $\omega_1 := -SRF_1 \cdot \omega_{roll}$

$$\theta_c(v, \omega_1, \phi\phi) = 81.787\,deg$$

typical draw shot spin:

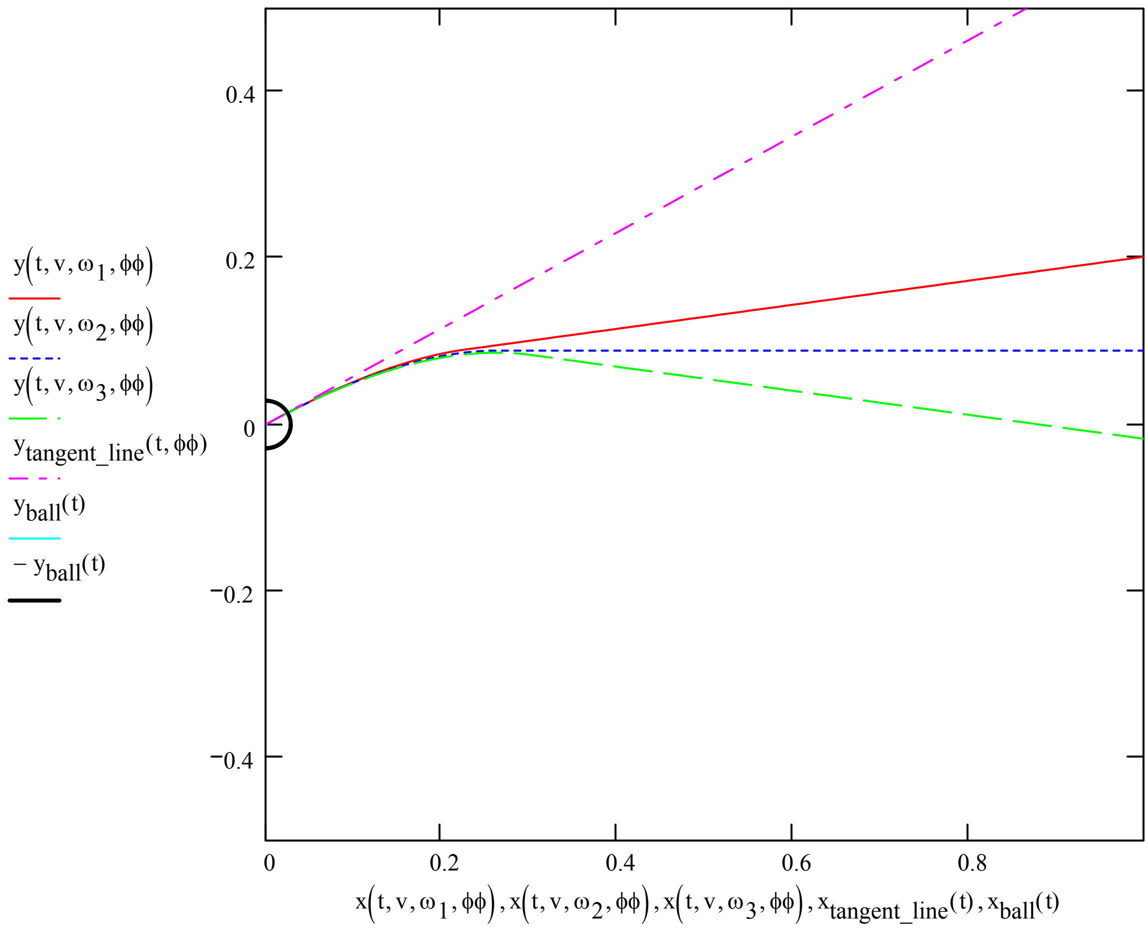
$SRF_2 := SRF\_TYP$ $SRF_2 = 0.625$ $\omega_2 := -SRF_2 \cdot \omega_{roll}$

$$\theta_c(v, \omega_2, \phi\phi) = 90\,deg$$

125% of typical draw shot spin (25% more than typical):

$SRF_3 := 1.25 \cdot SRF\_TYP$ $SRF_3 = 0.781$ $\omega_3 := -SRF_3 \cdot \omega_{roll}$

$$\theta_c(v, \omega_3, \phi\phi) = 98.213\,deg$$



y(t, v, ω1, ϕϕ)
y(t, v, ω2, ϕϕ)
y(t, v, ω3, ϕϕ)
ytangent_line(t, ϕϕ)
yball(t)
− yball(t)
0.4
0.2
0
−0.2
−0.4
0
0.2
0.4
0.6
0.8
x(t, v, ω1, ϕϕ), x(t, v, ω2, ϕϕ), x(t, v, ω3, ϕϕ), xtangent_line(t), xball(t)