technical proof

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TP 3.8
Effective target sizes for fast shots into a corner pocket at different angles

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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See the general notation presented in TP 3.5

See the specific notation and figure presented in TP 3.7

Ball radius

$$R := 1.125$$

Pocket-specific parameters

p: mouth width α: wall angle R_{hole}: hole radius b:shelf depth to hole

$$p := 4.58125 \quad \alpha := 7\cdot deg \quad R_{hole} := 2.75 \quad b := 1.125$$

Size of left portion of pocket target area assuming near miss of near point

$$s_{left\_point}(p, r, \theta) := \frac{p}{2}\cdot\cos(\theta) - r$$

$$D(p,\alpha,b,\theta) := \frac{1}{\sin(\theta + 5\cdot\alpha)}\cdot\left(\frac{p}{2}\cdot\cos(\alpha) - b\cdot\sin(\alpha) - R\right)$$

$$A(p,\alpha,b,\theta) := \frac{1}{\sin(\theta + 3\cdot\alpha)}\cdot\left(-\frac{p}{2}\cdot\cos(\theta + 4\cdot\alpha) + b\cdot\sin(\theta + 4\cdot\alpha) - D(p,\alpha,b,\theta)\cdot\sin(2\cdot\theta + 10\cdot\alpha) + R\cdot\cos(\theta + 3\cdot\alpha)\right)$$

$$B(p,\alpha,b,r,\theta) := \frac{1}{\sin(2\cdot\theta + 2\cdot\alpha)}\cdot(A(p,\alpha,b,\theta)\cdot\sin(\theta + 3\cdot\alpha) - r\cdot\cos(2\cdot\theta + 2\cdot\alpha) + R\cdot\cos(\theta + 3\cdot\alpha))$$

$$\gamma_1(\theta) := 90\cdot deg - \theta \qquad \gamma_2(\alpha,\theta) := \gamma_1(\theta) - 2\cdot\alpha$$

$$\gamma_3(\alpha,\theta) := -\gamma_2(\alpha,\theta) + 2\cdot\alpha \qquad \gamma_4(\alpha,\theta) := \gamma_3(\alpha,\theta) + 2\cdot\alpha$$

Maximum angle where ball can still be pocketed after near miss of near point

$$poly(p,\alpha,b,r,\theta) := p\cdot\cos(\alpha) - R - B(p,\alpha,b,r,\theta)\cdot\sin(\theta + \alpha) - r\cdot\cos(\theta + \alpha)$$

$\theta := 45\cdot deg$ initial guess for root function

$$\theta_{maximum}(p,\alpha,b,r,\theta) := root(poly(p,\alpha,b,r,\theta),\theta)$$

Size of left portion of pocket target area assuming inner wall contact at point

$$s_{left\_point\_wall}(p, \alpha, \theta) := \frac{p}{2} \cdot \cos(\theta) - R \cdot \cos(\theta - \alpha)$$

Critical angle where inner wall contact barely results in pocketing of ball

$$\theta := 45 \cdot deg$$

$$r_{critical}(p, \alpha, \theta) := (p \cdot \cos(\theta) - R \cdot \cos(\theta + \alpha))$$

$$\theta_{critical}(p, \alpha, b, \theta) := -root\left(poly\left(p, \alpha, b, r_{critical}(p, \alpha, \theta), \theta\right), \theta\right)$$

Size of left portion of pocket target area assuming inner wall contact

$$r := R$$

$$r_{left\_wall}(p, \alpha, b, \theta) := root(poly(p, \alpha, b, r, -\theta), r)$$

$$s_{left\_wall}(p, \alpha, b, \theta) := -\frac{p}{2} \cdot \cos(\theta) + r_{left\_wall}(p, \alpha, b, \theta)$$

$$\theta_{max} := \theta_{maximum}(p, \alpha, b, R, \theta) \qquad \theta_{max} = 59.841\ deg$$

$$\theta_c := \theta_{critical}(p, \alpha, b, \theta) \qquad r_{critical}\left(p, \alpha, -\theta_c\right) = 2.231 \qquad \theta_c = -52.226\ deg$$

$$s_{left}(\theta) := \begin{cases} s_{left\_point}(p, R, \theta) & \text{if } \theta \geq \alpha \\ s_{left\_point\_wall}(p, \alpha, \theta) & \text{if } \theta_c \leq \theta < \alpha \\ s_{left\_wall}(p, \alpha, b, \theta) & \text{otherwise} \end{cases}$$

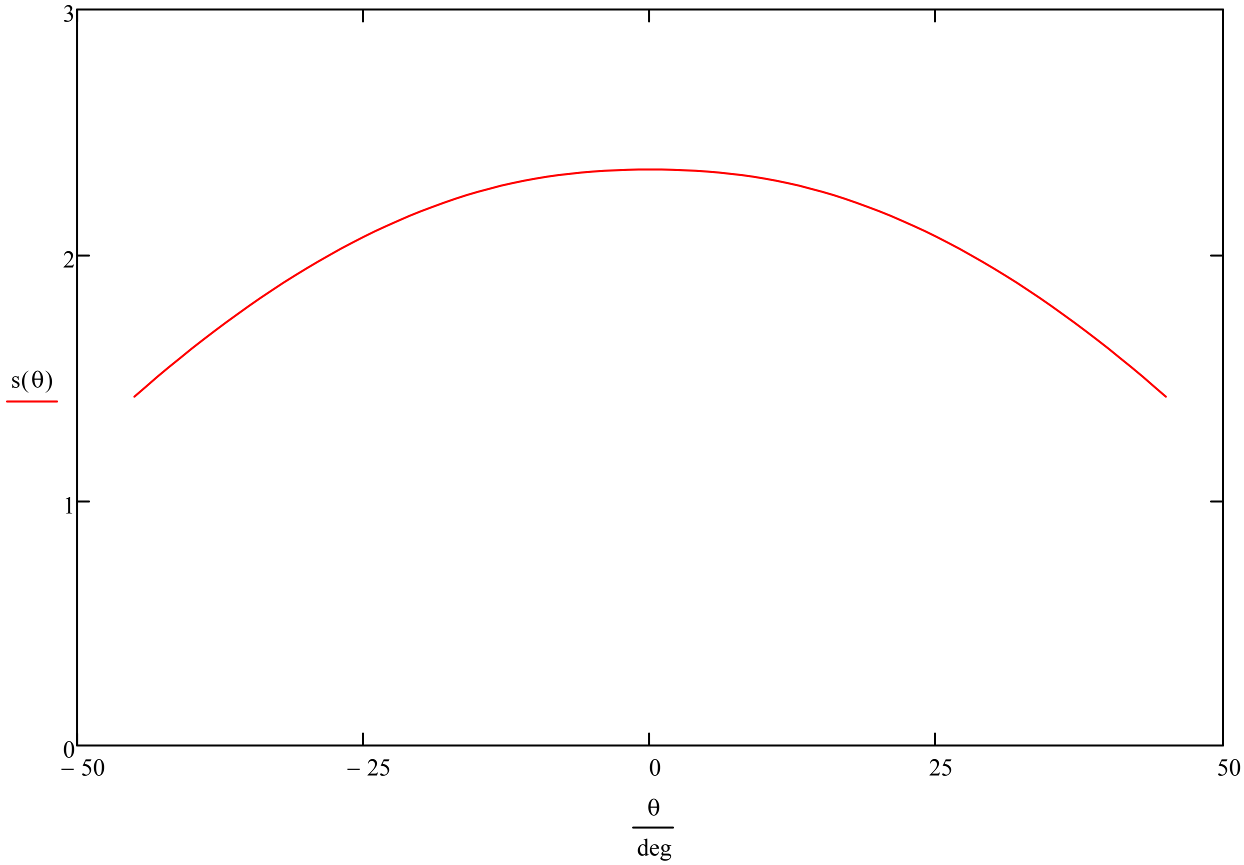
$$s_{right}(\theta) := s_{left}(-\theta)$$

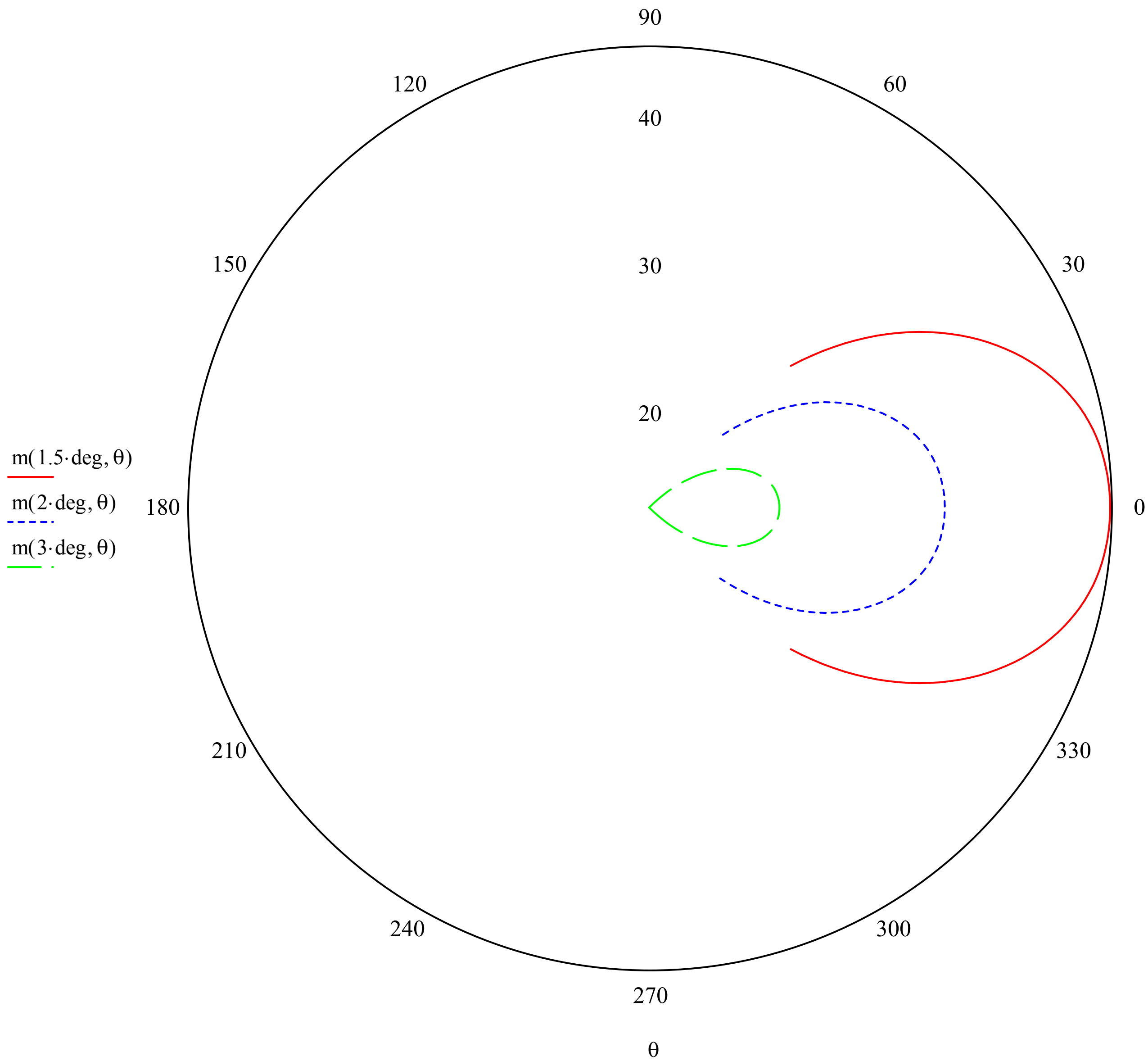
$$s(\theta) := s_{left}(\theta) + s_{right}(\theta)$$

$$offset(\theta) := \frac{s_{right}(\theta) - s_{left}(\theta)}{2}$$

$$m(\Delta\theta, \theta) := \frac{1}{2 \cdot \tan(\Delta\theta)} \cdot s(\theta)$$

$\theta := -45 \cdot deg, -44 \cdot deg .. 45 \cdot deg$




90
60
120
40
30
150
30
20
m(1.5·deg, θ)
m(2·deg, θ)
m(3·deg, θ)
180
0
210
330
240
300
270
θ