technical proof		technical proof

TP 3.7
Effective target sizes for fast shots into a side pocket at different angles

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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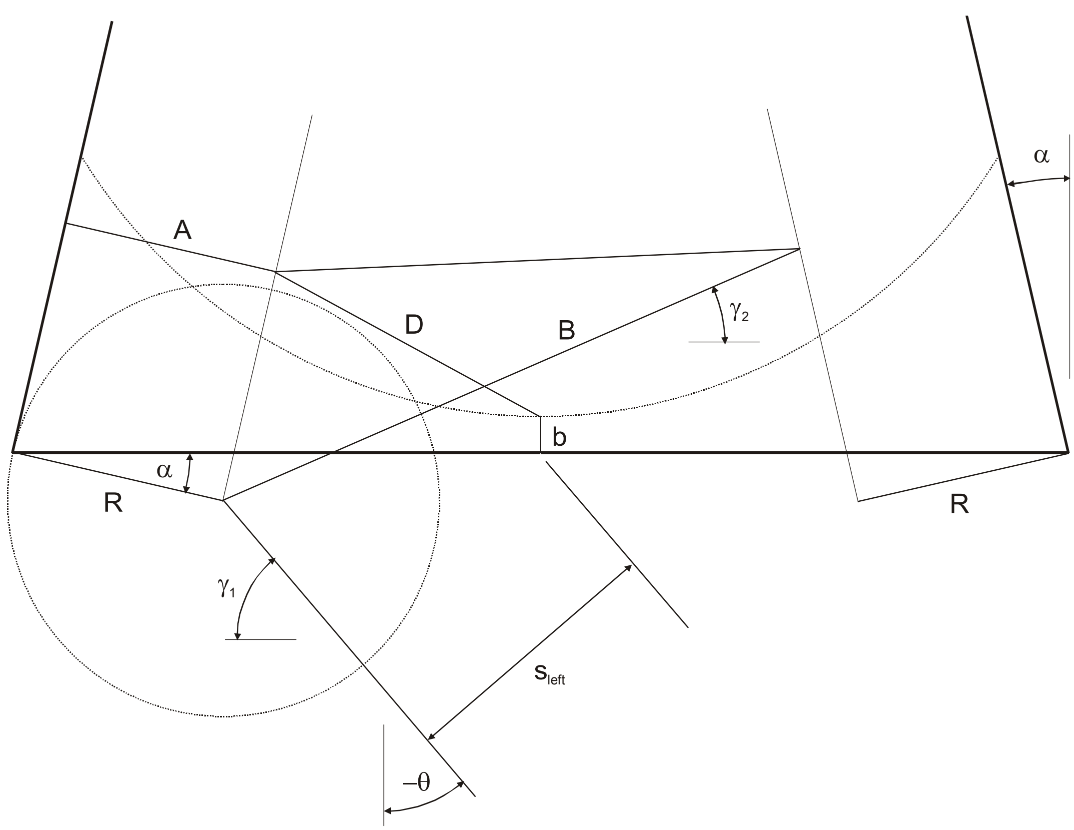
See the general notation presented in TP 3.5

Assumption: the ball will be pocketed if it rebounds at least to the center of the pocket hole rim after three wall rattles, assuming equal angles of approach and rebound on each wall (see figure below)



Ball radius

$R := 1.125$

Pocket-specific parameters

p: mouth width α: wall angle R_{hole}: hole radius b:shelf depth to hole

$p := 5.0625$ $\alpha := 14 \cdot deg$ $R_{hole} := 3$ $b := 0.1875$

Size of left portion of pocket target area assuming near miss of near point

$$s_{left\_point}(p, r, \theta) := \frac{p}{2} \cdot \cos(\theta) - r$$

$$D(p, \alpha, b, \theta) := \frac{1}{\sin(\theta + 5 \cdot \alpha)} \cdot \left(\frac{p}{2} \cdot \cos(\alpha) - b \cdot \sin(\alpha) - R \right)$$

$$A(p, \alpha, b, \theta) := \frac{1}{\sin(\theta + 3 \cdot \alpha)} \cdot \left(-\frac{p}{2} \cdot \cos(\theta + 4 \cdot \alpha) + b \cdot \sin(\theta + 4 \cdot \alpha) - D(p, \alpha, b, \theta) \cdot \sin(2 \cdot \theta + 10 \cdot \alpha) + R \cdot \cos(\theta + 3 \cdot \alpha) \right)$$

$$B(p, \alpha, b, r, \theta) := \frac{1}{\sin(2 \cdot \theta + 2 \cdot \alpha)} \cdot (A(p, \alpha, b, \theta) \cdot \sin(\theta + 3 \cdot \alpha) - r \cdot \cos(2 \cdot \theta + 2 \cdot \alpha) + R \cdot \cos(\theta + 3 \cdot \alpha))$$

$$\gamma_1(\theta) := 90 \cdot deg - \theta \qquad \gamma_2(\alpha, \theta) := \gamma_1(\theta) - 2 \cdot \alpha$$

$$\gamma_3(\alpha, \theta) := -\gamma_2(\alpha, \theta) + 2 \cdot \alpha \qquad \gamma_4(\alpha, \theta) := \gamma_3(\alpha, \theta) + 2 \cdot \alpha$$

Maximum angle where ball can still be pocketed after near miss of near point

$$poly(p, \alpha, b, r, \theta) := p \cdot \cos(\alpha) - R - B(p, \alpha, b, r, \theta) \cdot \sin(\theta + \alpha) - r \cdot \cos(\theta + \alpha)$$

$\theta := 45 \cdot deg$ initial guess for root function

$$\theta_{maximum}(p, \alpha, b, r, \theta) := root(poly(p, \alpha, b, r, \theta), \theta)$$

Size of left portion of pocket target area assuming inner wall contact at point

$$s_{left\_point\_wall}(p, \alpha, \theta) := \frac{p}{2} \cdot \cos(\theta) - R \cdot \cos(\theta - \alpha)$$

Critical angle where inner wall contact barely results in pocketing of ball

$$\theta := 45 \cdot deg$$

$$r_{critical}(p, \alpha, \theta) := (p \cdot \cos(\theta) - R \cdot \cos(\theta + \alpha))$$

$$\theta_{critical}(p, \alpha, b, \theta) := -root\left(poly\left(p, \alpha, b, r_{critical}(p, \alpha, \theta), \theta\right), \theta\right)$$

Size of left portion of pocket target area assuming inner wall contact

$$r := R$$

$$r_{left\_wall}(p, \alpha, b, \theta) := root(poly(p, \alpha, b, r, -\theta), r)$$

$$s_{left\_wall}(p, \alpha, b, \theta) := -\frac{p}{2} \cdot \cos(\theta) + r_{left\_wall}(p, \alpha, b, \theta)$$

$\theta_{max} := \theta_{maximum}(p, \alpha, b, R, \theta)$ $\theta_{max} = 50.688\,deg$

$\theta_c := \theta_{critical}(p, \alpha, b, \theta)$ $r_{critical}(p, \alpha, -\theta_c) = 3.358$ $\theta_c = -36.387\,deg$

$$s_{left}(\theta) := \begin{cases} s_{left\_point}(p, R, \theta) & \text{if } \theta \geq \alpha \\ s_{left\_point\_wall}(p, \alpha, \theta) & \text{if } \theta_c \leq \theta < \alpha \\ s_{left\_wall}(p, \alpha, b, \theta) & \text{otherwise} \end{cases}$$

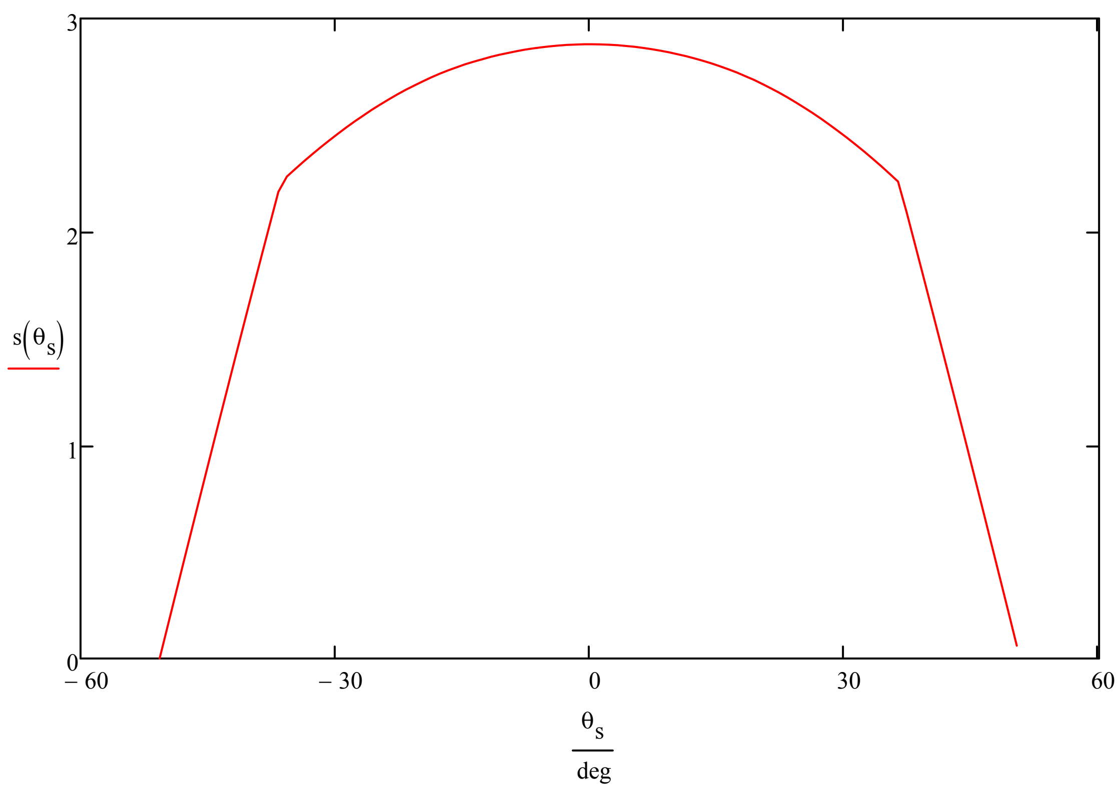
$s_{right}(\theta) := s_{left}(-\theta)$

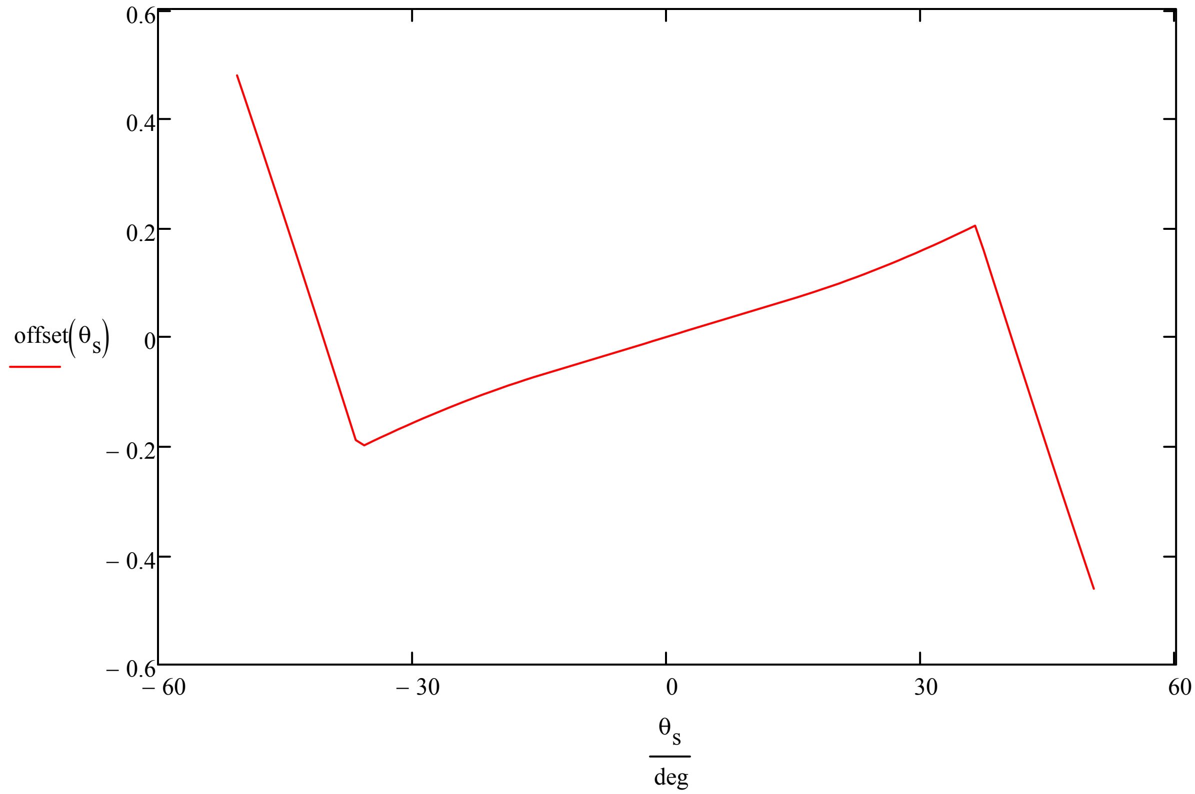
$s(\theta) := s_{left}(\theta) + s_{right}(\theta)$

$$offset(\theta) := \frac{s_{right}(\theta) - s_{left}(\theta)}{2}$$

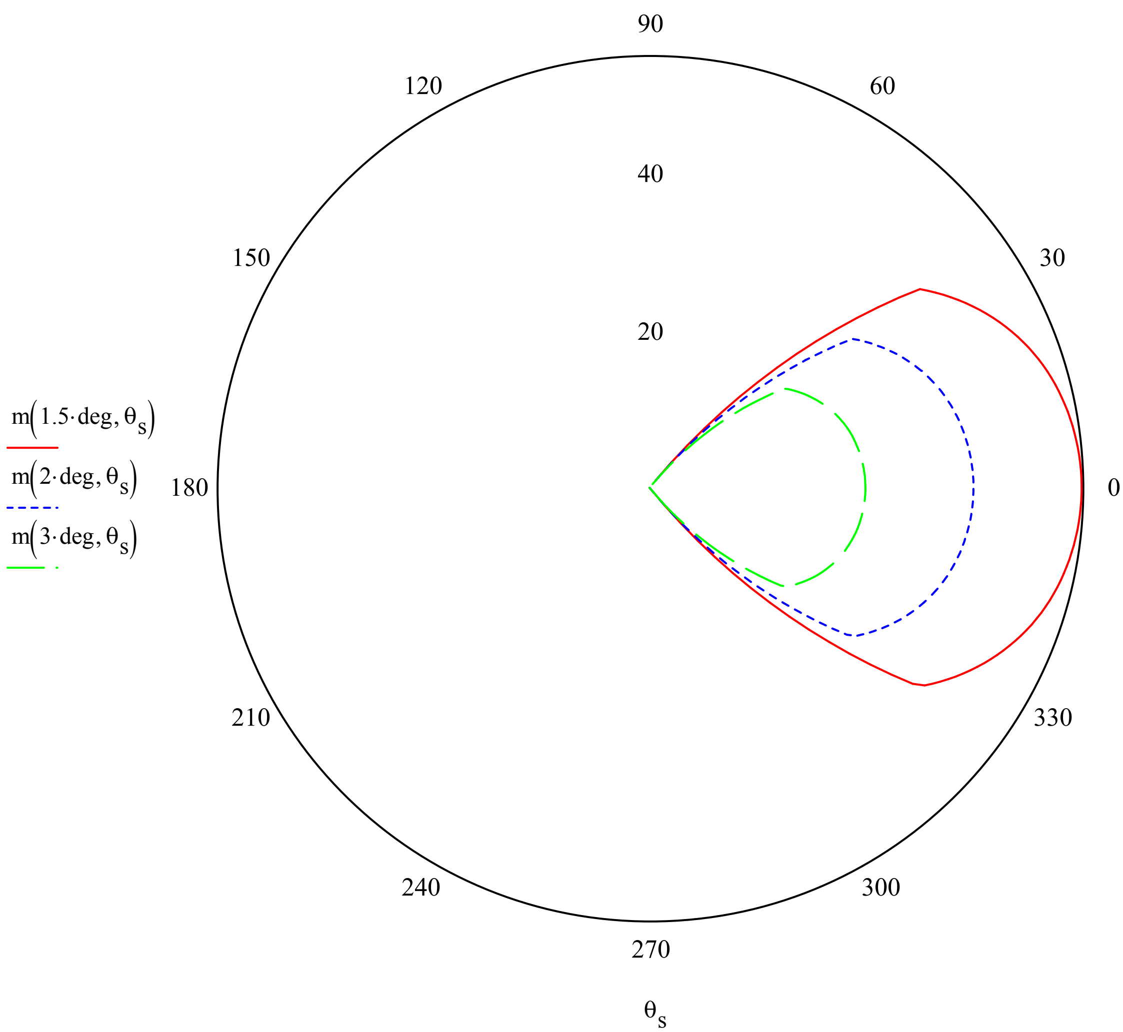
$$m(\Delta\theta, \theta) := \frac{1}{2 \cdot \tan(\Delta\theta)} \cdot s(\theta)$$

$\theta_s := -\theta_{max}, -\theta_{max} + 1 \cdot deg .. \theta_{max}$





0.6
0.4
0.2
0
− 0.2
− 0.4
− 0.6
− 60
− 30
0
30
60
offset(θs)
θs / deg



90
120
60
40
150
30
20
180
0
210
330
240
300
270
m(1.5·deg, θs)
m(2·deg, θs)
m(3·deg, θs)
θs