http://billiards.colostate.edu/physics/Alciatore_pool_physics_article.pdf

"Pool and Billiards Physics Principles by Coriolis and Others"

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Abstract

The purpose of this paper is to provide illustrations and explanations of many important pool and billiards physics principles. The goal is to provide a single and complete resource to help physics instructors infuse billiards examples into their lectures. The main contributions of Coriolis in his 1835 billiards physics book are presented along with other more recent developments and experimental results. Also provided are numerous links to pool physics references, instructional resources, and online video demonstrations. Technical derivations and extensive experimental results are not included in the article, but they are all available and easy to find online with the references and links provided.

key words: billiards, pool, physics, Coriolis, collision, friction, squirt, swerve, throw

I. INTRODUCTION

Pool (pocket billiards) is a great physics-teaching tool. It involves many physical principles including conservation of momentum and energy, friction, elastic and inelastic collisions, translational and rotational equations of motion, solid mechanics, vibrations, etc. Also, practically all students have either played or watched pool before, so they can relate to and get excited about pool examples, especially if the physics understanding might actually help them play better. Furthermore, because the playing surface of a pool table is ideally flat and the balls are ideally perfectly round and homogeneous spheres of equal mass, and ball collisions are nearly elastic and nearly friction-free, equations written for ball trajectories can actually be solved analytically, with only a few idealized assumptions.

In 1835, Gaspard-Gustave Coriolis wrote a comprehensive book presenting the physics of pool and billiards. Coriolis was not only a great mathematician and physicist ... he was also an avid billiards enthusiast. Coriolis' billiards physics book has not been very widely read because it was written in French, and an English translation has become available only recently (in 2005 by David Nadler [1]). There has also been many technical papers and online material published over the years expanding on pool physics knowledge [2-5]. In this article, I want to give an overview of many of the important and useful principles that can be used as examples in physics classes. To keep this paper of reasonable length, many of the technical derivations are provided online. I also plan to write more papers in the future that will delve more into some of the technical details and experimental results related to some of the principles.

The paper begins with some basic pool terminology, including important effects that come into play when sidespin is used (i.e., when the cue ball is struck left or right of center). Then I summarize many of the important principles discovered by Coriolis in the early 1800s. Finally, I present several topics for which I have done much work over the years: the 90º and 30º rules, cue ball “squirt” or “deflection,” and friction “throw” effects.

Supporting narrated video (NV) demonstrations, high-speed video (HSV) clips, and technical proofs (TP) referenced throughout the article can be accessed and viewed online at **http://billiards.colostate.edu**. The reference numbers used in the article (e.g., NV 3.1 and TP A.6) help you locate the resources on the website; and in the online article, the references are active links.

II. TERMINOLOGY

Figure 1 illustrates most of the important terms used to describe pool shots. The *cue stick* hits the *cue ball (CB)* into an *object ball (OB)*. After collision, the CB heads in the *tangent line* direction and the OB heads in the *impact line* (AKA *line of centers*) direction. The key to aiming pool shots is to be able to visualize the *ghost ball (GB)* target. This is the where the CB must be when it collides with the OB to send the OB in the desired direction, which is along the line connecting the centers of the GB and OB (see **NV 3.1** and **NV 3.2** for demonstrations). The *cut angle* is the angle between the original CB direction (i.e. the *aiming line*) and the final OB direction (i.e., the *impact line*).

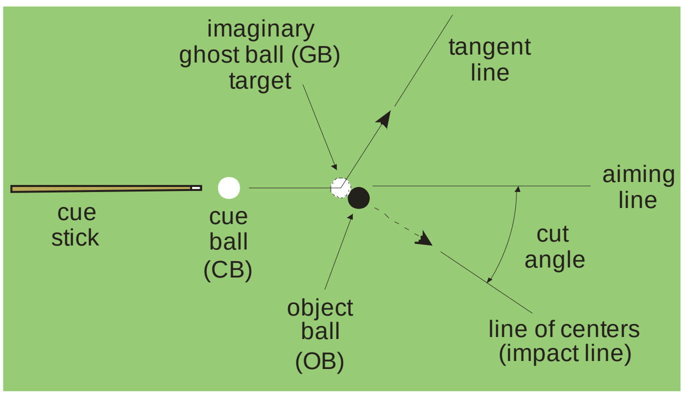

Figure 1 Pool terminology

Figure 1 is what happens ideally. Unfortunately, in real life, several non-ideal effects come into play. As shown in **Figure 2**, when using sidespin (also known as “english”), where the CB is struck to the left or right of center, the CB *squirts* away from the aiming line (see **NV 4.13** and **NV A.17**), *swerves* on its way to the OB (see **NV 4.14** and **NV 7.12**), and *throws* the OB off the impact line direction on its way to the target (see **NV 4.15**, **NV 4.16**, and **NV A.21**). If squirt, swerve, and throw didn't exist, pool would be a much easier game to master, but the physics wouldn't be as interesting.

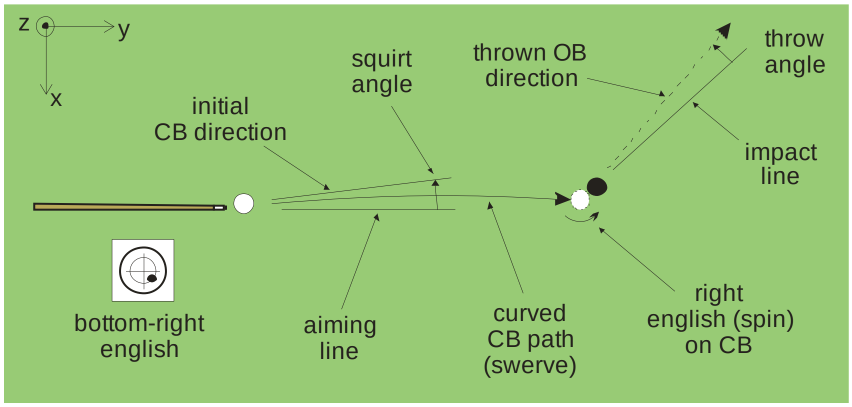

Figure 2 Non-ideal sidespin effects

Squirt and throw will be described in detail in Sections V and VI. Swerve is caused by the fact that the cue stick is always “elevated” some (i.e., the back end of the cue is higher than the tip end of the cue) to clear the rails bordering the table. Because of this, when you hit the CB off center, the ball acquires two spin components (see **Figure 3**) in addition to any top or bottom spin. One component is sidespin (about the z axis) caused by the moment created by the cue’s horizontal component of impact force (F_y) about the vertical (z) axis of the CB. American pool players refer to this component as “english;” interesting, the British refer to it as “side.” Pure sidespin has no effect on the path of the CB until it hits a rail cushion (see **NV 4.10** and **NV 4.11**). The other spin component, about the horizontal aiming-line (y) axis, is called *massé spin*. It is caused by the downward component of the cue stick’s impact force (F_z), which creates a moment about the y axis. **HSV A.127** shows a good example of the direction and effect of massé spin. The component of the friction force between the CB and cloth caused by the massé spin component is perpendicular to the CB’s direction of motion; hence, the CB’s path will be curved. With more cue stick elevation, the effect of the massé spin is greater causing the CB’s path to curve more (e.g., see **NV 7.11**, **NV B.41**, and **NV B.42**).

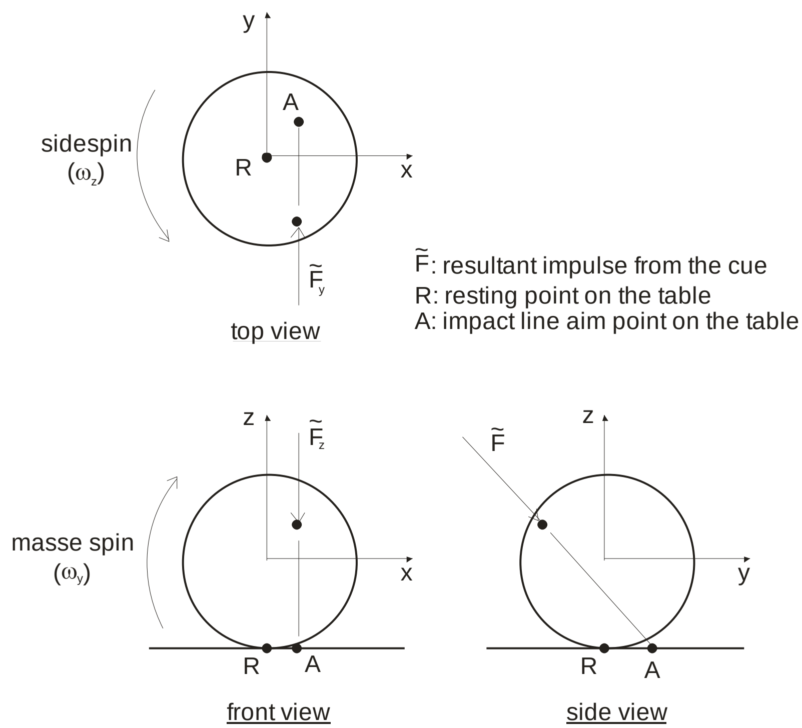

Figure 3 English and masse spin components

III. CORIOLIS, THE MATHEMATICIAN, PHYSICIST, AND ... BILLIARDS EXPERT

Below is a concise and illustrated summary of some of the important pool physics discoveries in Coriolis' 1835 book [1]. Additional information can be found in my series of articles describing and illustrating Coriois' work [6].

1. *The curved path followed by a CB after impact with an OB, due to top or bottom spin, is always parabolic.*

   Figure 4 *shows the effect of both bottom spin and topspin. If the CB had no spin, it would travel in the tangent-line direction (per the 90° rule in Section IV). When the CB deflects off the OB with any spin (e.g., a rolling CB would have topspin), the spin axis remains close to its original direction immediately after impact. This is a result of conservation of angular momentum, assuming for now friction between the balls is a small effect (which is the case, as shown in* ***TP A.6****). The spin now creates a friction force between the ball and cloth that has a component perpendicular to the direction of travel (see Figure 3), which causes the CB's path to curve. Because the magnitude of the sliding friction force is nearly constant [7], the magnitude of acceleration will also be nearly constant. It turns out that the relative velocity vector defining the direction and magnitude of the sliding motion, and therefore the friction force vector, also does not change direction during the sliding (see the derivation of Equation 10 in* ***TP A.4****). The relative slip speed gradually slows to zero and remains zero thereafter (i.e., the cue ball starts rolling without slipping at a certain point and continues to roll in a straight line, gradually slowing due to rolling resistance). Because the friction force vector, and therefore the cue ball acceleration, are constant in both magnitude and direction, the cue ball trajectory will be parabolic, just as with any constant acceleration motion (e.g., projectile motion), until the sliding ceases, in which case the CB heads in a straight line.* ***TP A.4*** *contains the full derivation.* ***HSV A.76*** *contains infrared super-slow motion video of example billiard shots clearly showing parabolic traces on the cloth (caused by heat generated by friction created by the spinning and sliding ball).* ***Figure 5*** *shows a still image from the video clearing showing a parabolic "hot" trace on the cloth. Also notice the "hot spots" caused by the spinning CB hopping several times after being struck with a downward stroke. In the video, you can clearly see how the sliding friction force direction remains constant as the "hot diameter" develops, intensifies, and persists on the CB.*

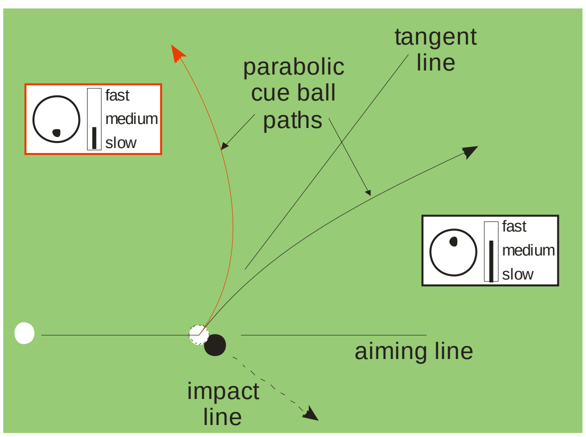

Figure 4 Parabolic CB paths

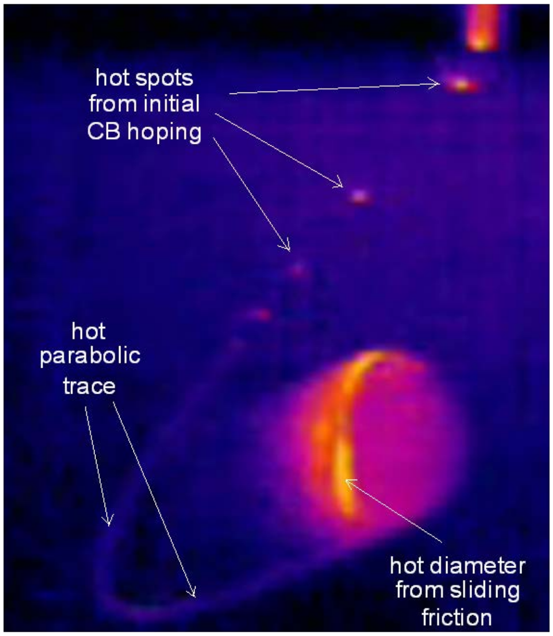

Figure 5 Parabolic CB paths (Courtesy of www.bskunion.at)

2. *To safely achieve maximum sidespin, the point of contact of the cue tip with the CB should be half a ball radius off center (see* ***Figure 6****).*

Obviously, the farther you hit the CB off center, the more sidespin you impart. Although, if the tip offset exceeds the half-ball-radius (0.5R) amount, a miscue, where the cue tip slides off the CB during impact, is very likely (see ***NV 2.1*** *and* ***HSV 2.1****). Needless to say, a miscue is undesirable in a game situation, because the cue ball does not head in the intended direction. Alciatore (in part IV of [6]) performed some high-speed video analysis to experimentally determine maximum effective tip offsets possible (without miscue) with typical pool equipment. The largest measured effective offset was about 0.55R. As shown in* ***TP 2.1****, 0.55R and 0.5R correspond to required coefficients of friction between the tip and ball of about 0.66 and 0.58 respectively. To achieve these values, the leather tip must be properly shaped and textured and have chalk applied properly, but all serious pool players do this, so the 0.5R limit is appropriate for most equipment.*

In Coriolis' book [1], there are actually several different analyses involving tip offset. In one analysis (a similar analysis can be found in TP A.30), he shows that for a theoretical offset greater than 0.6R, the amount of english will be reduced because the cue tip will not slow down enough after initial impact and it will stay in contact with the cue ball for a while; although, he assumed the cue stick is rigid with no deflection and no cue ball squirt. The claimed result is that the cue tip would rub on the spinning cue ball creating friction after impact, which would reduce the amount of spin. However, high-speed video analysis *(described in part IV of [6]) shows that at large offsets, the cue tip deflects away from the cue ball and does not remain in contact. Regardless of the various analyses concerning tip offset limitations, the miscue limit defined by the maximum possible coefficient of friction appears to be controlling factor with typical pool equipment.*

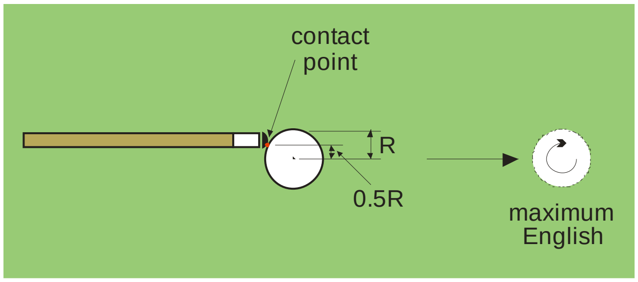

Figure 6 Contact point offset for maximum sidespin

3. *With a massé shot, where the CB is struck from above off-center, the final path of the CB will be in a direction parallel to the line drawn between the initial base point of the CB and the aiming point on the table.*

*The technique is illustrated in **Figure 7**. The final direction of the CB path is parallel to line RA, which connects the original CB resting point (point R) to the aiming point on the cloth (point A). Using the letters shown in the diagram, with "B" indicating the CB contact point, I refer to the Coriolis massé aiming system as the "BAR" method ("B" for ball, "A" for aim, and "R" for resting point). This technique can be useful when trying to aim massé shots, where you need to curve the CB around an obstacle ball (e.g., see **NV 7.11**, **NV B.41**, and **NV B.42**). The detailed physics and math behind the BAR massé aiming method and resulting ball paths can be found in **TP A.19**. The math, physics, and geometry is fairly complex; but, conceptually, the result makes sense based on the fact that the initial spin axis (which is perpendicular to the RA direction) creates a friction force component that remains in the RA direction (per the parabolic trajectory arguments above) until the sliding ceases and rolling begins.*

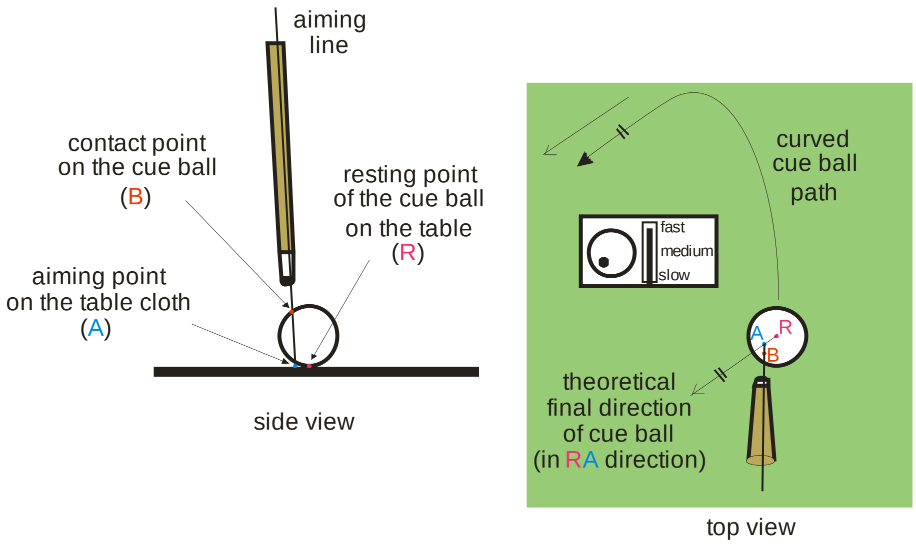

Figure 7 "BAR" massé shot aiming method

4. *For a CB with natural roll, the largest deflection angle the CB can experience after impact with an OB is 33.7°, which occurs at a cut angle of 28.1° (see Section IV for illustrations and a lot more detail on why this is useful).*

IV. 90º AND 30º RULES

The most important goal in pool is sending the OB into the desired pocket (i.e., "making a shot"). The second most important goal is knowing where the CB will go so you can easily make the next shot. **Figure 8** illustrates one of the most important principles of pool related to this: the *90° rule* (see **NV 3.4**, **NV 3.5**, and **TP 3.1**). It states that when the CB strikes an OB with no topspin or bottom spin, the two balls will always separate at 90°. In other words, the CB will head exactly in the tangent line direction and persist along this line. This is true regardless of the cut angle (see Figure 8). Note – in the remainder of this paper, the CB and OBs will be assumed to all have equal mass. This assumption is usually very close to reality at a pool table. The CB can sometimes be heavier (e.g., with some coin operated tables, where a mechanism under the table can automatically sort the CB from the OBs based on larger size and/or larger weight). The CB is more typically slightly lighter than the OBs because it experiences more collisions, some abrasion during tip impact, and abrasion with sliding on the cloth, all of which cause wear and reduced mass.

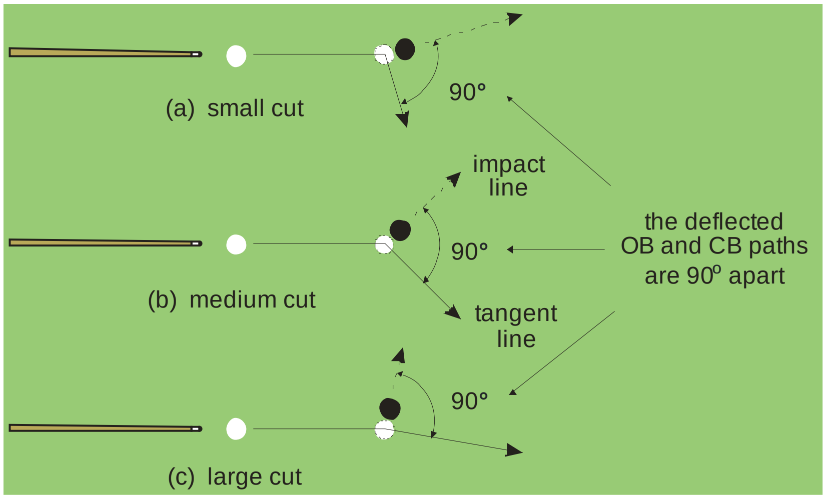

Figure 8 The 90° rule

The 90° rule is a direct result of the principles of conservation of energy and conservation of linear momentum (see **TP 3.1**). All of the CB's momentum in the impact line direction (see Figure 8b) gets transferred to the OB, assuming an elastic collision. The result is that the final paths of the balls are perpendicular. This is what happens ideally. In real life, there is a small amount of energy loss and retention of impact-line momentum when the CB hits the OB. This effect is often quantified with a *coefficient of restitution (COR)*. The COR for perfect (ideal) balls would be 1, representing 100% conservation of energy. Typical pool balls have a COR closer to 0.93. Therefore, the 90° rule is actually closer to the 85° rule (see **TP A.5**).

The 90° rule, assuming elastic collisions, applies exactly only for a *stun shot*, where the CB is sliding without topspin or bottom spin at impact with the OB. The one exception to the 90° rule is when the CB hits the OB perfectly squarely, with no cut angle. In this case, the CB stops completely, transferring all of its speed to the OB. This is called a *stop shot* (see **NV 3.6**, **HSV 3.1**, and **HSV 3.2**). Notice how in **HSV 3.1**, to achieve a stop shot, one must often hit the CB below center to create bottom spin, which gradually slows, so the CB has no spin at impact with the OB.

Figure 9 illustrates the physics involved with a stop shot. As the CB slides along the cloth, the friction force between the ball and cloth creates a torque about the ball's center that gradually reduces (decelerates) the bottom spin (see Figure 9b and **HSV 3.1**). If the CB were to continue to slide, the friction force would continue to change the CB's spin, slowly building up forward roll, as shown in Figure 9a (see also: **NV B.10**). Note how the CB actually decelerates (as indicated qualitatively by the relative lengths of the straight arrows in the figure) as the topspin builds, because the sliding friction force is in the opposite direction as the ball's motion (see the friction arrows in the figure). Figure 9c shows what happens if the CB starts out with over-spin, which implies more topspin than the natural roll amount. The spin rate decelerates (as indicated qualitatively by the relative shortening of the curved arrows) until rolling develops (see **HSV B.46**). In this case, the CB actually accelerates because the sliding friction force is in the same direction as the ball's motion (see the figure). When the CB is rolling, there is no longer any sliding friction, and the CB continues to roll naturally until the ball slows to a stop (due to rolling resistance) or hits something (e.g., another ball or a rail). If the CB is struck with the cue tip at the *center of percussion (COP)*, the ball will roll immediately. For a pool ball, the COP is at 70% of the ball's height above the table surface, for a level cue (see **TP 4.2**). To get over-spin (e.g., as in Figure 9c), the CB must be struck above this height, which is risky due to likelihood for miscue. Over-spin also occurs when a rolling CB hits an OB (e.g., see **HSV B.26**).

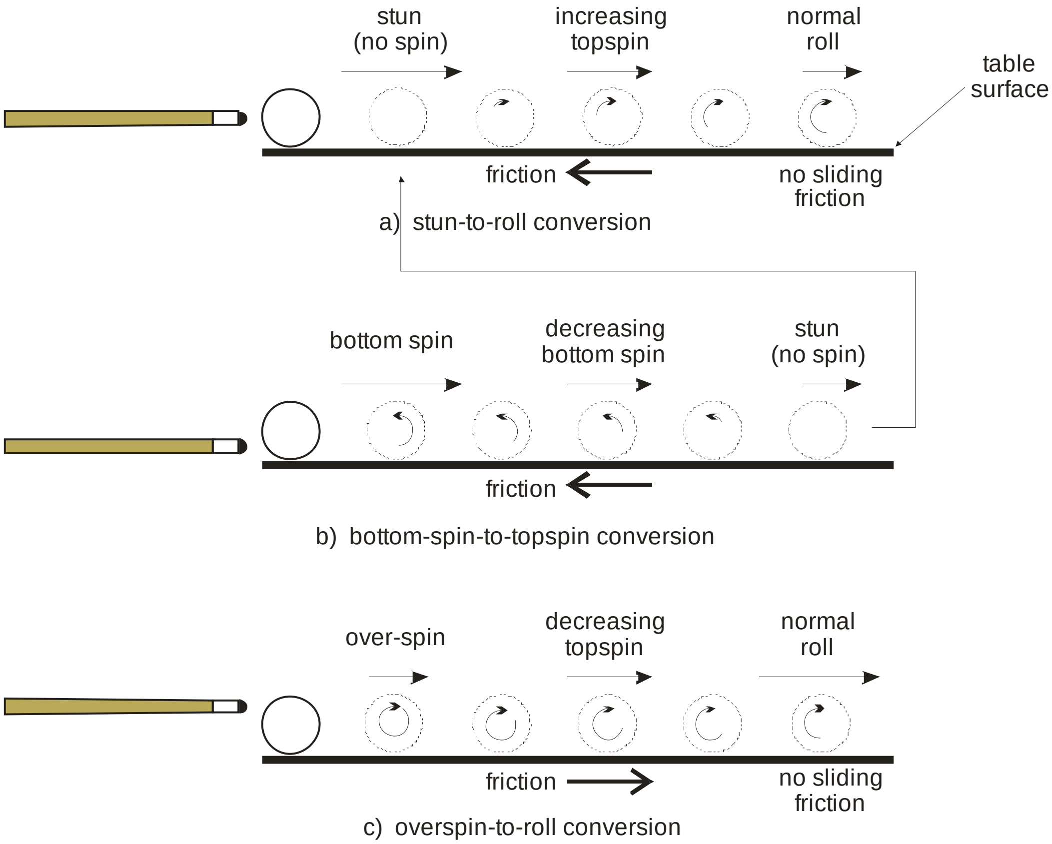

Figure 9 Conversion of vertical-plane spin to normal roll (side view)

As described above, with most pool shots, the CB will be rolling by the time it reaches the OB. The exception is where bottom spin and/or fast speeds are being used. When the CB is rolling, the 90° rule no longer applies. What applies instead is governed by what I call the *30º rule* (see **Figure 10**, **NV 3.8**, **NV 3.9**, **NV 3.10**, **NV B.43**, and **NV B.44**). It states that when a rolling CB hits an OB close to a *half-ball hit* (see **Figure 11**), the CB will deflect approximately 30° away from its initial aiming line. In **TP 3.3**, I show the detailed physics and math behind the rule, along with a modern proof for the numbers in Coriolis' 4th conclusion (see Section III). Wallace and Schroeder [8] and Onoda [9] also present the supporting technical background for the 30° rule.

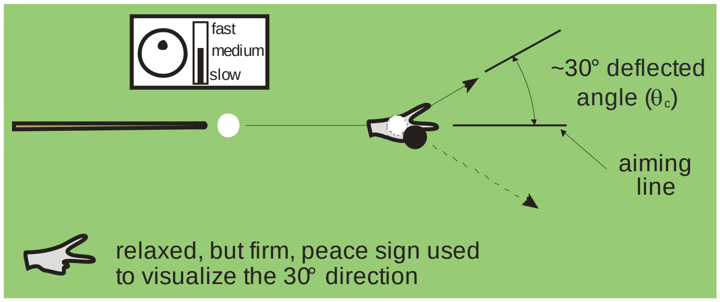

Figure 10 30° rule

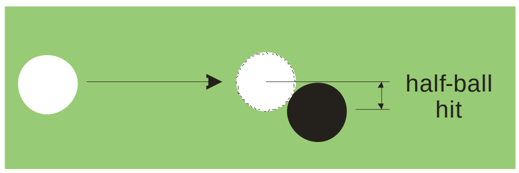

Figure 11 Half-ball hit

As illustrated in Figure 10 and demonstrated in **NV 3.8**, you can use your hand to help visualize the 30° CB direction. As shown in **Figure 12**, if you form a relaxed but firm V-shape (peace sign or victory symbol) with your index and middle fingers, the angle between your fingers will be very close to 30°. **NV 3.8**, **NV 3.9**, and **NV B.44** show how to use the *Dr. Dave peace sign* in practice. If you point one of the fingers in the aiming line direction, the other finger will indicate the direction the CB will travel after impact.

Fortunately for pool players, as shown in **Figure 13**, the 30° rule applies over a wide range of ball-hit fractions (see **TP 3.3**). The center of the range is the half-ball hit, but the CB deflection is very close to 30° for ball-hit fractions as small as 1/4 and as large as 3/4. The exact CB deflection angle θ_c as a function of cut angle ϕ is given by (see Equation 38 in **TP A.4**):

$$\theta_c = \tan^{-1}\left(\frac{\sin(\phi)\cos(\phi)}{\sin^2(\phi)+\frac{2}{5}}\right) \tag{1}$$

The derivation is fairly involved and complication; but, qualitatively, the shape in Figure 13 makes sense. For a small ball-hit fraction (i.e., large cut angle), the spin axis of the ball after impact is close to the spin axis of the original rolling CB, and very little translational speed is lost in the collision, so there is not much sliding between the CB and table after impact, and the CB's direction doesn't change very much (i.e., $\theta_c \approx 0$). For a large ball-hit fraction (i.e., small cut angle), much of the CB's translational speed is transferred to the OB, and the spin that remains on the CB after impact, which is almost all of the original roll topspin amount, creates sliding friction and accelerates the ball forward (as in Figure 9c above). So in this case, the CB's original direction also doesn't change very much (i.e., $\theta_c \approx 0$). In the center of the range, both the post-impact translational motion (along the tangent line) and the acceleration due to friction caused by the CB's rotation (as with Figures 4 and 7 above) combine to cause a deflection angle in between the original aiming line and the tangent line. The maximum CB deflection angle happens to occur close to a half-ball hit (30º cut angle), as derived in **TP A.4**. As determined by Coriolis (see also: **TP 3.3**), the actual value is approximately a 0.53-ball hit, corresponding to a 34º cut angle.

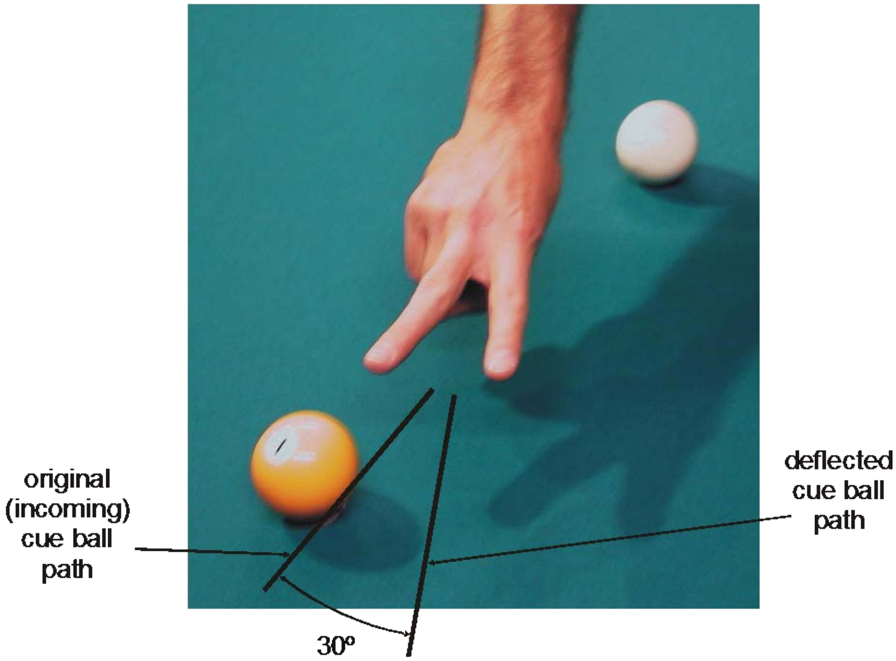

Figure 12 Using your hand to visualize the 30° rule CB paths

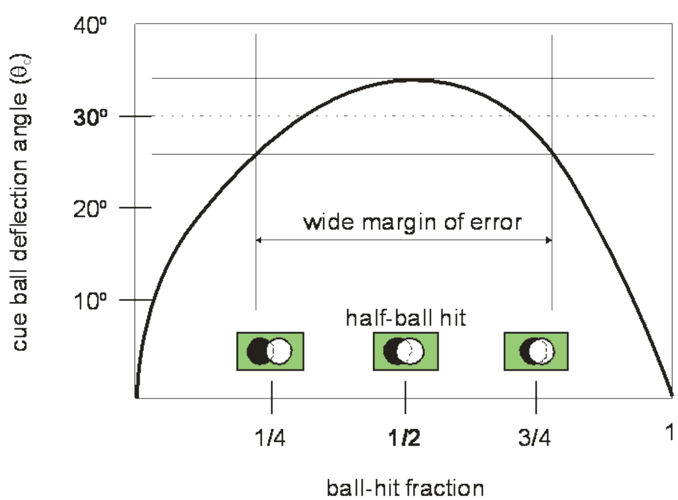

Figure 13 Large margin of error for 30° rule

Figure 14 illustrates the ball-hit-fraction range, and corresponding cut angles, to illustrate the wide range of shots for which the 30° rule applies. With a 1/4-ball hit (see Figure 14a), the center of the CB is aimed outside of the object-ball edge such that the projected cue-ball-path passes through 1/4 of the OB. With a 1/2-ball hit (see Figure 14b), the center of the CB is aimed directly at the edge of the OB such that the projected cue-ball-path passes through 1/2 of the OB. With a 3/4-ball hit (see Figure 14c), the center of the CB is aimed inside of the object-ball edge such that the projected cue-ball-path passes through 3/4 of the OB. These three cases cover a fairly large range of cut angles between 14° and 49° (see **TP A.23**). Most pool shots are in this range.

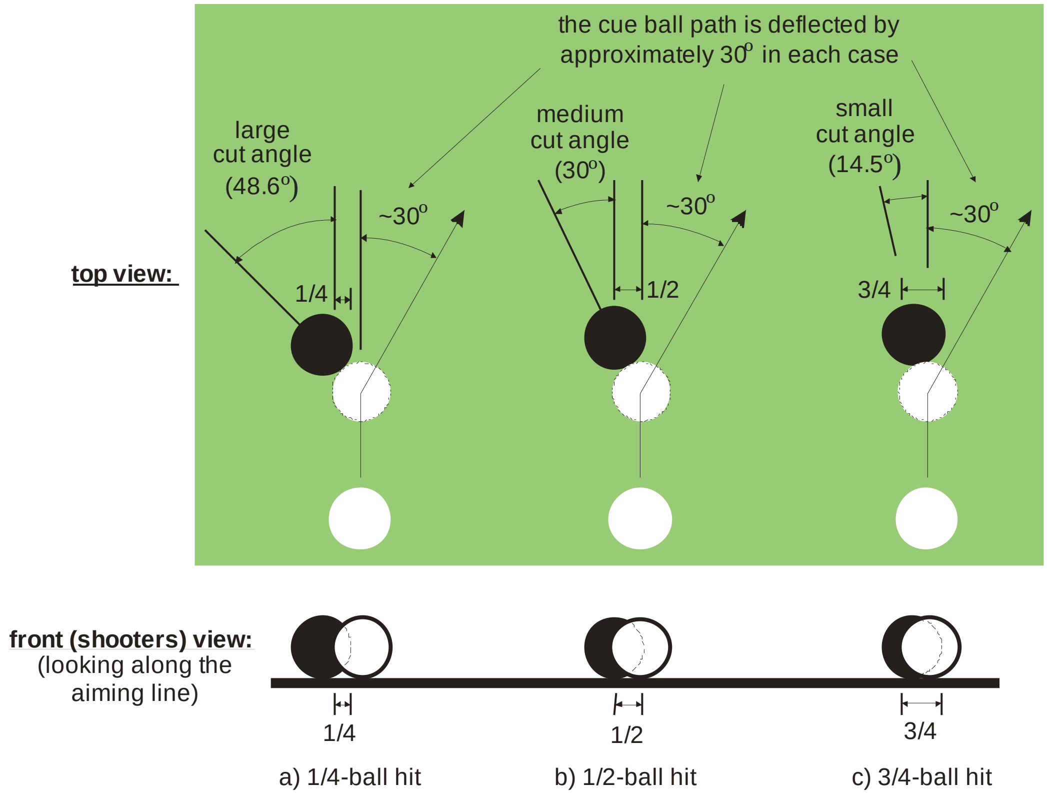

Figure 14 Various ball-hit fractions

The 30° rule and peace sign technique are extremely useful in practice. This is a great example where a little knowledge of the physics of the game can be a big help. The rule can be used to detect possible scratches (where you pocket the CB by mistake), plan carom shots (where you deflect one ball off another into a pocket), plan ball avoidance or break-up shots, and strategically plan CB position as you run a rack of balls. Examples of all of these types of shots can be found on my website (e.g., see **NV 3.7**, **NV 3.10**, **NV 7.2**, **NV 7.3**, **NV 7.4**, **NV A.1**, and **NV B.46**).

V. SQUIRT

Figure 15 shows selected stills from a high-speed-video clip that illustrates the physics behind *squirt*. The full clip, which was filmed at 2000 frames/sec, can be viewed online at **HSV A.76a**. Still “a” is just before contact. Stills “b” through “e” represent 0.0015 sec during which the tip is in contact with the ball. This is actually quite typical for most pool shots ... the cue tip is in contact with the CB only for about 1-2 thousandth of a second, regardless of shot speed. In still “f” the tip hasn’t fully recovered from the compression yet as the CB is separating. Still “g” is after separation. The line and arc appearing in each still mark the initial cue stick and CB positions. Notice how much the cue tip deflects away (down in the diagram) from its original line of action. Also notice how much the cue tip deforms (e.g., see still “d”). Those leather tips, which haven’t changed much in 200 years, are quite resilient. The black arrows in still “c” illustrate the effect that causes squirt. While the tip is in contact with the ball, the ball starts rotating. This rotation (counterclockwise in the diagram) pushes the cup tip down a little during contact. Because the end of the shaft has mass, it takes force to move the end of the shaft down as the ball rotates. An equal and opposite reaction on the CB is what causes the CB to squirt away from the cue aiming line.

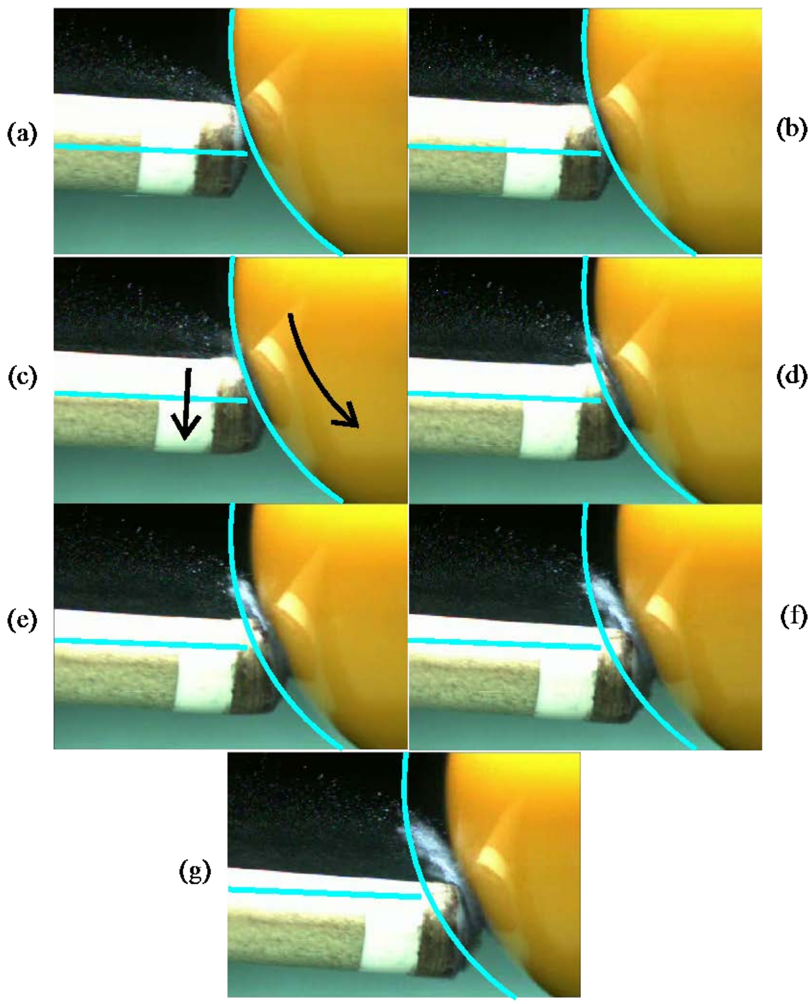

Figure 15 Close-up of cue tip impact during an off-center hit

Shepard [10] and **TP A.31** provide the complete derivation relating squirt angle to tip offset and the "effective endmass" of the cue, which is a function of geometry and material properties of the end of the shaft. The *endmass* relates to how far the transverse elastic wave travels down the shaft (from the tip) during the brief contact time between the tip and ball. In other words, the "endmass" is a measure of how much effective mass at the end of the cue comes into play due to transverse (sideways) motion of the tip during contact with the CB. Experiments [11, 12] have suggested that only the last 15-20 cm (6-8 in) of typical cue shafts contribute to "endmass." This is why cue manufactures have been successful with reducing squirt by using a smaller tip and shaft diameter, using a smaller and lighter ferrule (the white plastic component between the tip and the shaft), and drilling out the end of the shaft, all to reduce the effective endmass [13]. It would be interesting to do more experiments to quantify and better model how mass and stiffness distribution affect endmass, and to measure the elastic wave speed and characterize the dependence of transverse stiffness, but we or others have not done this yet.

In the squirt analysis, the tip is assumed to remain in contact with the ball as the ball rotates (see **HSV A.76a** for visual evidence of how well a chalked leather tip "grabs" the ball); otherwise, a miscue results, where the tip slides off the CB and the CB heads in unpredictable directions. While the tip and ball are in contact, the velocity of the tip and ball are equal at the point of contact. This assumption, along with linear and angular impulse-momentum principles, is used in **TP A.31** to derive the following result:

$$\alpha = \tan^{-1}\left[\frac{\frac{5}{2}\frac{b}{R}\sqrt{1-\left(\frac{b}{R}\right)^2}}{1+\frac{m_b}{m_e}+\frac{5}{2}\left[1-\left(\frac{b}{R}\right)^2\right]}\right] \qquad (2)$$

where, as illustrated in **Figure 16**, α is the squirt angle, b is the amount of tip offset from the center (i.e., the tip contact point eccentricity), R is the ball radius (1 1/8 in = 28.6 mm), m_b is the mass of the CB (usually 6 oz = 170 gm), and m_e is the effective endmass of the cue (usually in the 5-15 gm range). The squirt angle is very close to a linear function of tip offset, provided miscues and partial miscues are not occurring. Cross [14] has verified this theoretical result with a series of experiments. The linear relation makes it easy to compensate for squirt when using sidespin [15]. Cross [14] also investigated how squirt angle changes with tip offset under conditions of cue tip slip, but this doesn't occur with non-miscue pool shots.

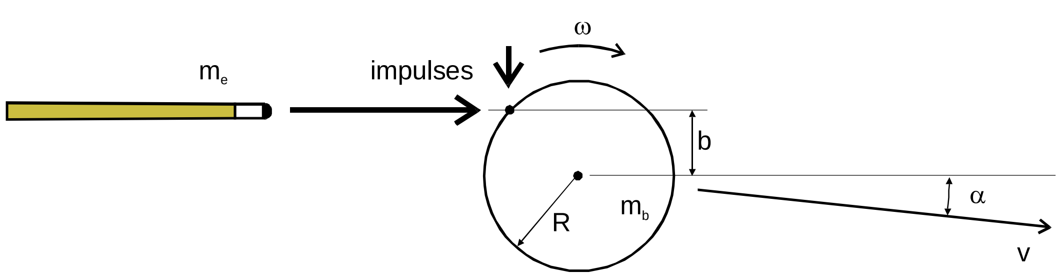

Figure 16 Squirt terminology

Figure 17 shows a squirt-testing robot students and I have designed and built to perform studies with the factors that influence squirt. The machine consists of a spring-loaded carriage on a linear rail that consistently and accurate delivers the cue at a desired speed and CB tip-contact point. The cue is held perfectly horizontal throughout the entire stroke (i.e., the cue is not "elevated") to eliminate CB swerve as a factor. This is an important feature because swerve can vary with cue elevation, ball speed, shot distance, and ball and cloth conditions.

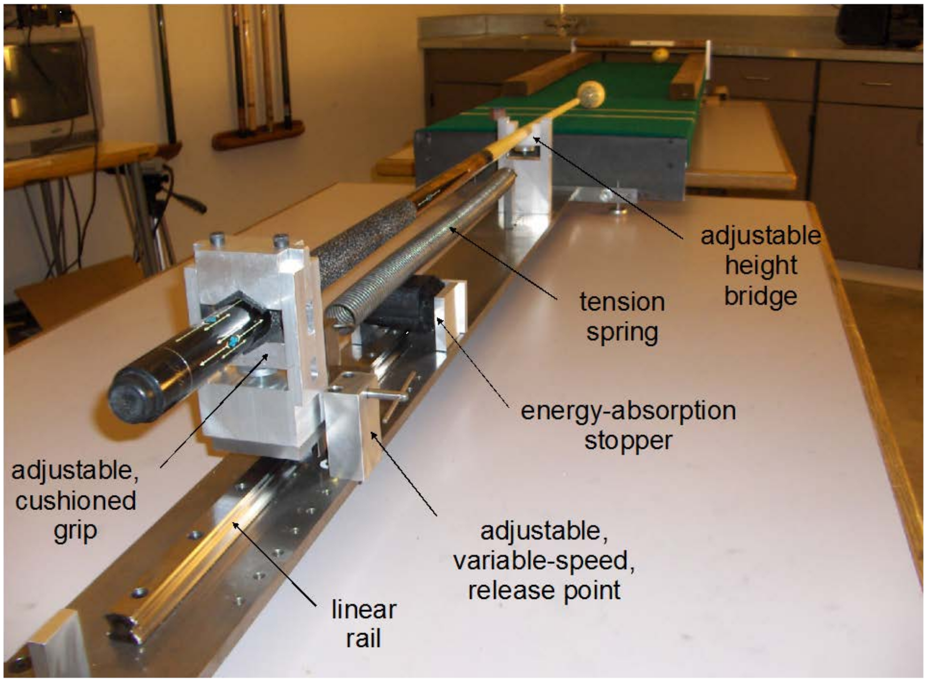

Figure 17 Cue squirt testing machine

Figure 18 shows the results of an experiment to show how end-mass affects a cue's squirt. In the experiment, a mass was added to the cue at different positions. The masses used in the experiment weighed 0.3 grams and 1.1 grams. The two curves show the end-mass effect very clearly. With more mass added to the end of the cue, and the closer the mass is to the tip, the more the cue ball will squirt. Also, if mass is added beyond a certain distance from the tip, it has no effect on the amount of squirt. For the cue tested, the distance was about 18 cm (7 in). Notice how the curves level out to the same squirt value beyond the 18 cm (7 in), regardless of the amount of the added mass.

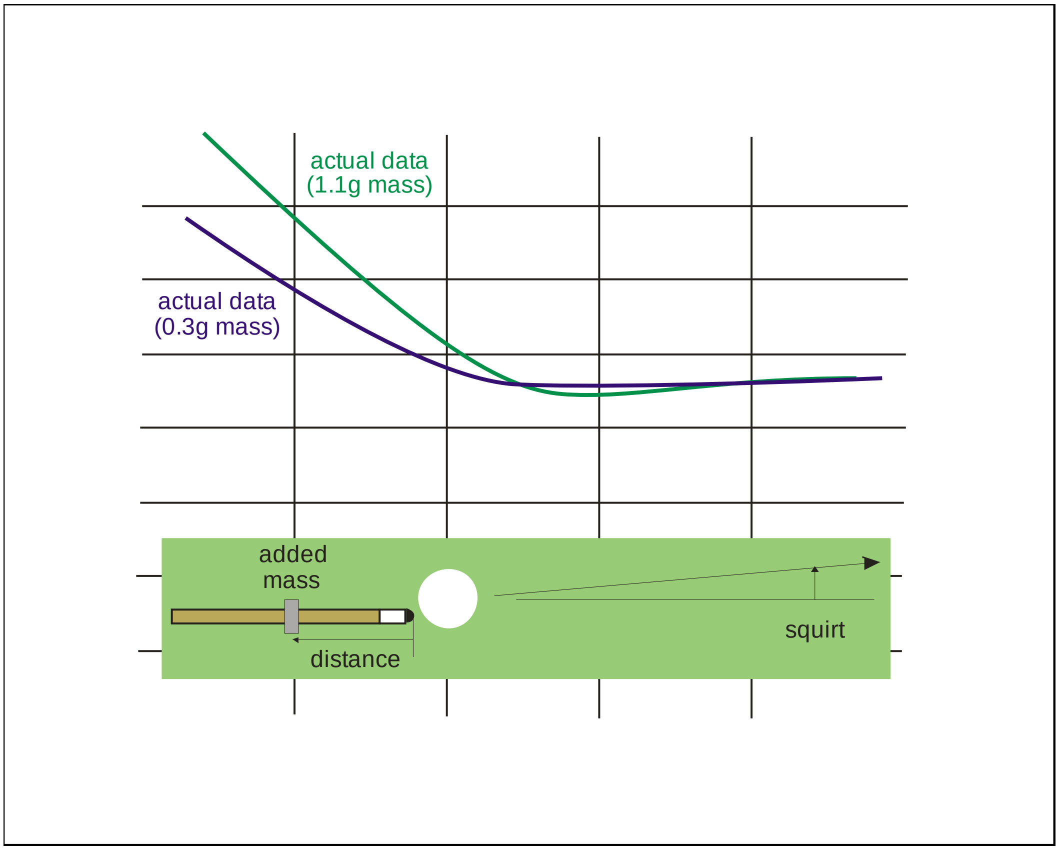

Figure 18 Squirt vs. position of added mass

One more thing to note in Figure 18 is how the squirt seemed to dip a little lower for the larger added mass, before flattening out. When we first saw this, we wondered if adding even more mass at this point would reduce the squirt even further. If that were the case, we might have discovered a new way to make a low-squirt cue. Unfortunately, adding more mass at that point did not cause additional decreases in squirt.

VI. THROW

Figure 19 illustrates all of the important terminology and physics concerning throw. Throw occurs any time there is relative sliding motion between the CB and OB surfaces at impact. In the figure, the relative motion is a result of CB sidespin; so the resulting throw would be called *spin-induced throw (SIT)*. Throw can also be caused by cut angle alone, in which case it is called *cut-induced throw (CIT)*. In Figure 19, the right (counterclockwise) sidespin creates a sliding friction force that pushes the OB to the left. This force is what creates the *throw angle*. The spin imparted to the OB is called *transferred spin*. In this case, the throwing force to the left creates clockwise (right) transferred spin on the OB. Spin transfer is an important effect with bank shots (e.g., see **NV A.21**).

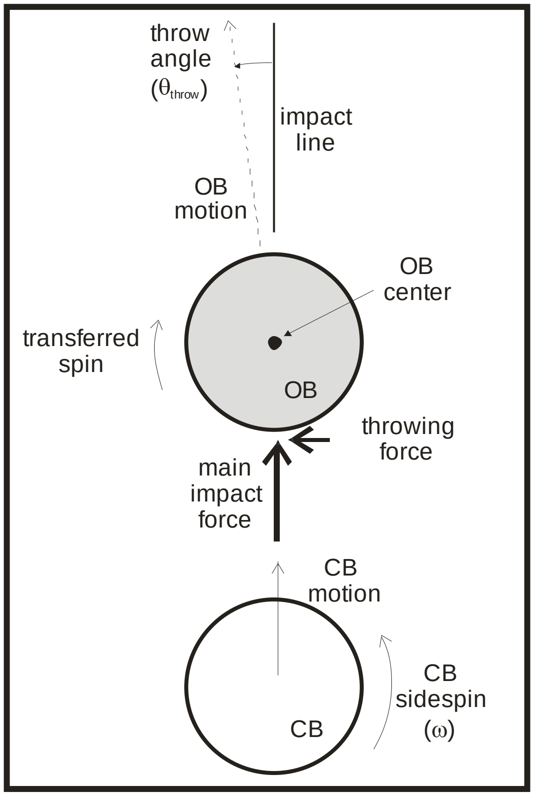

Figure 19 Throw and spin transfer terminology and physics

Throw angle θ_{throw} depends on the speed of the shot v, the cut angle ϕ, the amount and direction of CB spin ω, and the friction properties of the balls. The complete derivation can be found in **TP A.14**, and here is the main result:

$$\theta_{throw} = \tan^{-1}\left\{ \left[\min\left(\frac{\left(a_\mu + b_\mu e^{-c_\mu \sqrt{\left(v\sin(\phi) - R\omega_z\right)^2 + \left(R\omega_x \cos(\phi)\right)^2}} \right) v\cos(\phi)}{\sqrt{\left(v\sin(\phi) - R\omega_z\right)^2 + \left(R\omega_x \cos(\phi)\right)^2}},\quad 1/7 \right) \left(v\sin(\phi) - R\omega_z\right) \right] \Big/ v\cos(\phi) \right\} \tag{3}$$

where a_μ, b_μ, and c_μ are parameters used to characterize how dynamic sliding friction varies with the relative surface speed between the CB and OB at impact.

The 1/7 term in the "min" function in Equation 3 results from a requirement that relative sliding motion between the CB and OB cannot reverse direction during impact. In the limiting case, the relative velocity between the ball surfaces goes to zero during impact (i.e., sliding ceases during impact). The total possible friction impulse during impact is limited by this 1/7 factor (see **TP A.14**). Previous treatments of pool-ball throw in the literature (e.g., Chapter 4 in [4]), have not included the no-slip-reverse constraint in the friction model.

The exponential model for friction used in Equation 3 is based on experimental data from Marlow (p. 245 in [3]) and Alciatore (**TP B.3**). The coefficient of dynamic friction between pool balls is far from constant ... it is much lower at higher relative speeds between the ball surfaces.

The exponential model of friction in Equation 3 fits experimental data quite well (see **TP A.14** and **TP B.3**). Possible reasons for why the friction coefficient decreases with higher relative surface speed include: surface asperities might glance over each other (and not intermesh as easily) with a moving surface, and boundary layer effects due to entrapped and displaced air might have a larger effect at higher speeds.

Figure 20 shows an example plot generated from Equation 3. This particular plot shows how the amount of throw changes with cut angle for a stun shot (i.e., sliding CB in translation only) at various speeds (slow, medium, and fast). Useful conclusions one can observe from the figure include:

- For small cut angle shots (i.e., fuller hits), the amount of throw does not vary with shot speed, but increases with cut angle. The reason for this is the 1/7 factor described above. At small cut angles, sliding between the balls ceases during impact, and the coefficient of sliding friction has no effect on the total tangential impulse experienced during the collision.
- For larger cut angle shots (i.e., thinner hits), the amount of throw is significantly larger for slower speed shots as compared to faster speed shots. This is due to the lower coefficient of sliding friction resulting from the faster relative speed between the sliding ball surfaces during impact. If friction did not vary with speed (e.g., if a constant were used for μ in Equation 3), there would only be a single curve in Figure 20, and the right portion would be a horizontal, straight line.
- The amount of throw decreases a little at large cut angles, but not by much (especially for slower speed shots). Again, this is due to the sliding friction speed dependence. At larger cut angles, the relative speed of sliding is larger for a given ball speed.
- The maximum throw occurs at close to a half-ball hit (30º cut angle).

Experiments performed by Jewett [16] and Alciatore [17, 18] have verified many of these and other trends predicted by Equation 3. Jewett's data is shown in **Figure 21**. The exact throw values in Figure 20 cannot be directly compared to those in Figure 21 because the exact physical properties of the balls, and the precise speeds used for "slow," "medium," and "fast," are not available. However, the point of the figures is to show that the experimental data do exhibit the same trends predicted by the model. Other experiments (e.g., for follow and draw shots [18]) also verify trends predicted by the model.

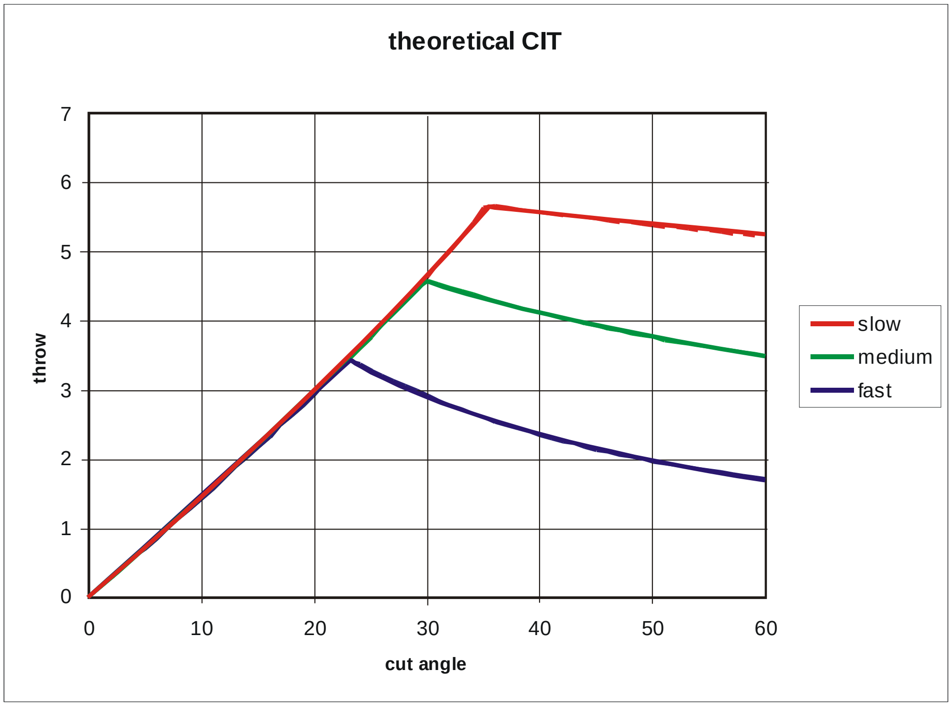

Figure 20 Stun shot collision-induced throw

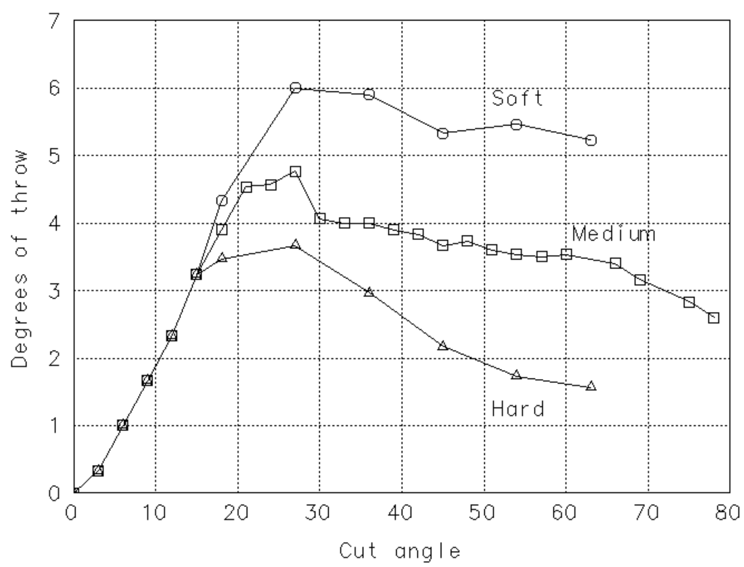

Figure 21 Experimental stun-shot cut-induced-throw data (Courtesy of Bob Jewett)

Numerous graphs and contour plots showing how throw varies for all types of shots can be found in **TP A.28** and **TP B.3**. Depending on the speed, angle, and type and amount of spin on the ball, the throw amount can vary significantly. Obviously, to play pool at a high level, one's aim must be compensated to account for throw variations with different types of shots.

VII. CONCLUSIONS

This article has presented a collection of pool and billiards principles that make for excellent class examples of many fundamental physics topics including friction, energy, momentum, inelastic and elastic collisions, kinematics, and equations of motion. I have used pool examples in many engineering and physics classes over the years, and students seem to enjoy and relate to them. Due to size constraints, I have not included derivations or much experimental data in the article. Instead, I have referenced online resources where the details can be found. I wanted this article to be comprehensive summary document with good illustrations and complete lists of online and print references for educators who want to use pool and billiards examples in their courses. In future articles, I plan to present more details concerning derivations and experimental results for some of the more interesting and complex topics (e.g., throw and squirt). As a final note, if you plan to assign any pool or billiards analysis questions for homework in any of your physics classes, there is a useful summary of typical values for all important physical parameters in [19].

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Note:
All of the Billiards Digest articles referenced above are also available at: http://billiards.colostate.edu