"The Amazing World of Billiards Physics"

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Note: Supporting narrated video (NV) demonstrations, high-speed video (HSV) clips, and technical proofs (TP) can be accessed and viewed online at http://billiards.colostate.edu. The reference numbers used in the article help you locate the resources on the website.

It's 1810, you're in a French prison, but because you are a model prisoner, you are given access to a pool table donated by a local aristocrat. Your cue stick is an old style "mace" that has a solid wooden tip, because the cue tip hasn't been invented yet. The wooden tip cracks and fails to make good contact with the cue ball anymore ... not that it ever really did. You decide to tear a small piece of leather off your shoe to attach to the cracked end of the mace to improve cue ball contact. All of the sudden, your accuracy and possible arsenal of shots improve dramatically.

Sounds like an unlikely story? Well, that's pretty much what happened to Francois Mingaud, the imprisoned Frenchman credited with inventing the leather cue tip. The leather tip allowed a cueist to put "*English*" on the cue ball, which is sidespin caused by hitting the cue ball off center. So a "Frenchman" invented billiards "English." (By the way, the British use the term "side" instead of "English" to refer to sidespin.) "English" allows the shooter to better control where the cue ball goes during a shot (e.g., see **NV 4.25**). The cue tip was a monumental invention for the pool world; and interesting, the equipment really hasn't changed much since. We still use wooden cue sticks with leather cue tips.

NV 4.25 – Positioning the cue ball at all spots on the table from an easy side pocket shot

normal video

Well, the main purpose of this article isn't to tell the story of the history of billiards equipment. Instead, we will look at the wonderful world of billiards physics and show how a little physics knowledge can help you be a better pool player. First, let's start with some terminology.

Terminology

"*Billiards*" is a term that generally refers to all cue sports. This includes pocket billiards, played on a table with pockets. American *pool* and British *snooker* are the two most common types of pocket billiards. 8-ball and 9-ball are the two most common pool games. Some billiards games are played on tables without pockets. The most common is *three-cushion billiards* played with only three balls. Before pool halls came along in America, three-cushion billiards was a very popular game. The term *pool* comes from its early history where wagering (like a football "pool") was often part of the game (and still is in many pool halls).

Figure 1 illustrates most of the important terms used to describe pool shots. The *cue stick* hits the *cue ball (CB)* into an *object ball (OB)*. After collision, the CB heads in the *tangent line* direction and the OB heads in the *impact line* (AKA *line of centers*) direction. The key to aiming pool shots is to be able to visualize the *ghost ball (GB)* target. This is the where the CB must be when it touches the OB to send the OB in the desired direction, which is along the line connecting the centers of the GB and OB (see **NV 3.1** and **NV 3.2** for advice on how to practice and improve your visualization and aiming skills).
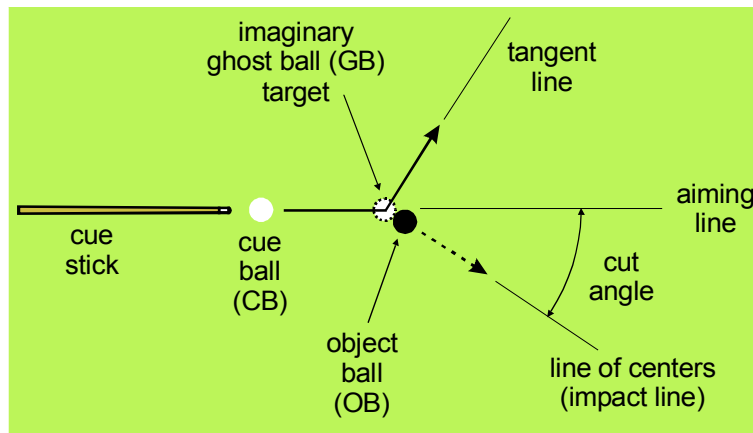

Figure 1 Pool terminology

NV 3.1 – Practicing contact point and ghost ball visualization
NV 3.2 – Using the cue stick to help visualize the impact and aiming lines

normal video

Figure 1 is what happens ideally, in the diagram of a book or magazine article. Unfortunately, in real life, several non-ideal effects come into play. As shown in **Figure 2**, when using sidespin, the CB *squirts* away from the aiming line (see **NV 4.13** and **NV A.17**), *swerves* on its way to the OB (see **NV 4.14** and **NV 7.12**), and *throws* the OB off the impact line direction on its way to the target (see **NV 4.15**, **NV 4.16**, and **NV A.21**). Squirt and swerve will be described below, and a whole section will be dedicated to throw. If you don't compensate your aim for squirt, swerve, and throw when playing pool (consciously or sub-consciously), you will never be a great player. After learning a little more about these effects, maybe you will know a little better why you might have missed some pool shots in the past; if not, at least you will have a few more excuses if you miss more in the future.
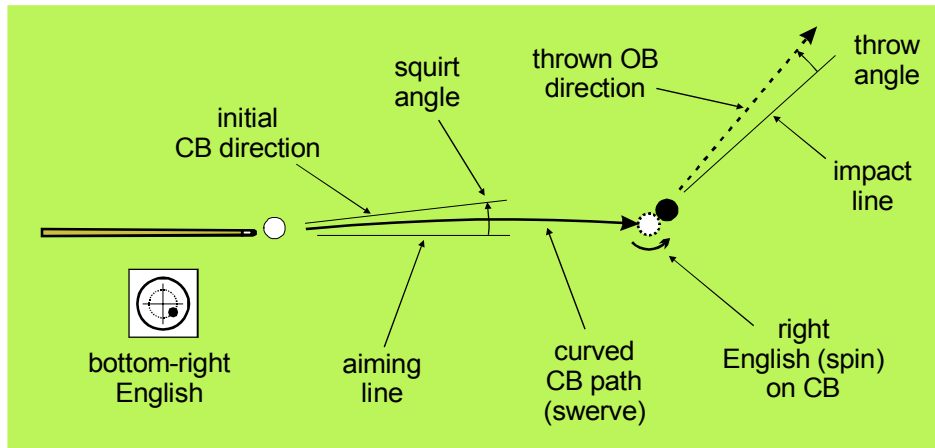
Figure 2 Non-ideal sidespin effects

normal video

    Figure 3 shows selected stills from a high-speed-video movie that illustrates the physics behind *squirt*. The full clip can be viewed, in super-slow-motion, at **HSV A.49**. First note the cue tip is in contact with the ball only between the 2nd and 3rd stills. The "3000FPS" at the bottom-right portion of the still indicates that the clip was filmed at 3000 frames per second. The elapsed time (being counted down) is shown on the 2nd line at the top-right of each still. The elapsed time between the 2nd and 3rd stills is only 0.001 second (see the timer in the 2nd line of text in the images). This is actually quite typical for most pool shot ... the cue tip is in contact with the CB only for about a thousandth of a second! The duration from the 1st to the last (6th) still is only 0.0127 second (a little more than a hundredth of a second).

    In **Figure 3**, note how much the cue stick deflects away from the CB as a result of the impact. This explains why the CB squirts in the opposite direction. Newton said that for every action, there is an equal and opposite reaction. If the cue stick deflects to one side, there must be force acting on the tip in that direction and on the CB in the opposite direction. So the force that causes the cue stick to deflect and vibrate with sidespin shots (e.g., see **HSV 4.4**) is the same force that causes the CB to squirt off its aiming line. The force is related to how much end mass the cue stick has. "Low-squirt" cue sticks (e.g., Predator's Z-shaft) have low end-masses to limit how much sideways force can be supported. With less mass, there is less force and, therefore, less squirt (i.e., the CB will go closer to the direction you are aiming).
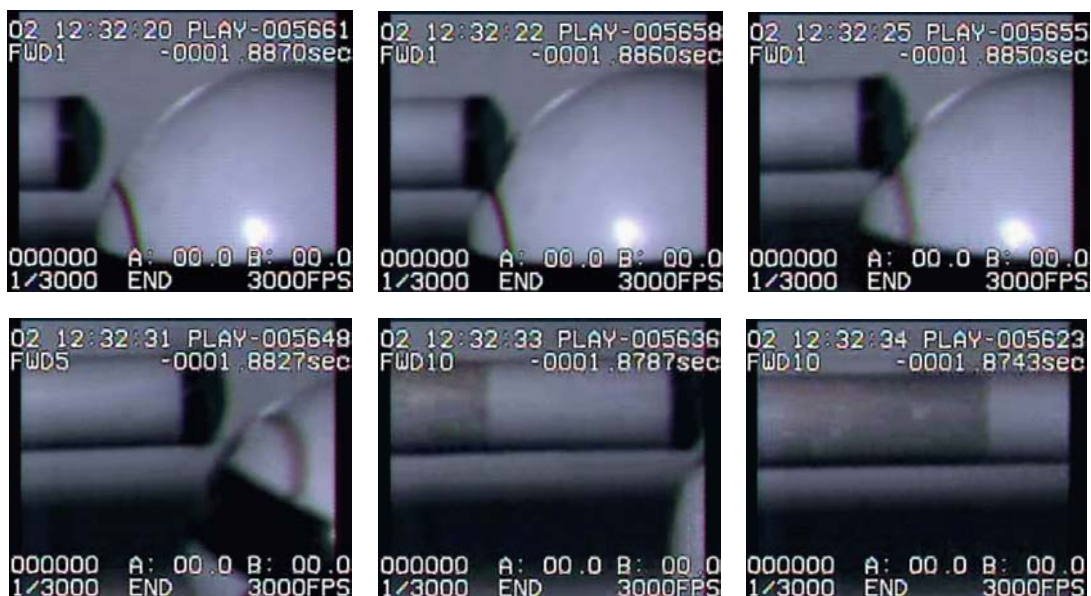
Figure 3 Close-up of cue tip impact during a large-offset follow shot

high-speed video

HSV A.49 – Follow shot with large offset, light grip, good follow-through, and fast speed
HSV 4.4 - Deflection (squirt) due to high-speed English

Swerve is caused by the fact that the cue stick is always elevated some (to clear the rails bordering the table). Because of this, when you hit the CB off center, the ball picks up two spin components. One component is sidespin caused by the moment arm of the cue tip about the vertical axis of the CB. Pure sidespin has no effect on the path of the CB until it hits a rail cushion (see **NV 4.10** and **NV 4.11**). The other spin component is called *massé spin*. It is caused by the downward component of the stick's impact force, which results from the downward angle of the cue stick. This spin is like the "body roll" of an airplane or the "Eskimo roll" of a kayak. **HSV A.127** shows a good example of the direction and effect of massé spin. The friction between the CB and cloth, caused by the massé spin component, is perpendicular to the CB's direction of motion; hence, the CB's path is curved. With more cue stick elevation, the amount of massé spin will be greater, and the CB's path will be more curved (e.g., see **NV 7.11**). Maybe you have seen some dramatic examples of this on TV if you have ever watched one of ESPN's Trick Shot Magic pool tournaments.

normal video

NV 4.10 – Effect of left English
NV 4.11 – Effect of right English
NV 7.11 – Large-curve massé shot

high-speed video

HSV A.127 – Massé shot

Coriolis, the mathematician, physicist, and ... billiards expert

In 1835, Gaspard-Gustave Coriolis wrote a comprehensive book presenting the physics of pool and billiards. This is the same Coriolis who discovered the "Coriolis" effect that accounts for the dynamics of artillery shells, hurricanes, and flushing toilets. (The toilets part isn't quite true, but it would take too long to explain.) Coriolis was not only a great mathematician and physicist ... he was also an avid billiards enthusiast. Coriolis' billiards physics book has not been very widely read because it was written in French, and an English translation has become available only recently (in 2005 by David Nadler [1]). Also, it is very difficult to read and understand by today's standards. I had the pleasure to review the manuscript a few years ago, and I was blown away by the thoroughness and informative nature of the book (maybe a more appropriate word is tomb). He had studied and totally figured out pretty much every physical principle that describes what happens at pool table. And the leather tip had only just been invented a few years before his time. Coriolis was probably not that good of a pool player himself – his mathematics and physics endeavors didn't leave him with much spare time – but luckily, he had pool-playing acquaintances that provided creative inspiration and provocative questions for his math and physics studies.

I was so inspired by Coriolis' work that I wrote a series of six articles for Billiards Digest magazine, presenting his billiards discoveries. This was quite the challenge because I had to attempt to present the work of this brilliant mathematician and physicist in a manner that could be understood by the readership of a billiards magazine, most of who do not have advanced technical degrees. I'll share a brief summary of some of Coriolis' results below. Much more information can be found in my July-December 2005 instructional articles ("Coriolis was Brilliant ... but he didn't have a High-speed Camera") available on my website.

Following is a brief summary of some of Coriolis' conclusions, which he backed up with theoretical and limited experimental studies.

1. *The curved path followed by a CB after impact with an OB, due to top or bottom spin, is always parabolic* (see **Figure 4** and **TP A.4**). This is useful to know when you are trying to predict where the CB will go for shots with different types of spin (e.g., see **NV 4.20** and **NV 4.21**). **Figure 4** shows the effect of both bottom spin (called a draw shot) and topspin (called a follow shot). If the CB had no spin, it would head in the tangent-line direction (per the 90° rule in the next section). With top or bottom spin, the axis of the spin is perpendicular to the direction of motion to begin with, so it results in no curving of the CB's initial path. After the CB deflects off the OB, the direction of motion changes, but the spin axis remains close to its original direction, as a result of the principle of conservation of angular momentum. The spin now creates a friction force between the ball and cloth that has a component perpendicular to the direction of travel. This results in the curving of the path, much like with a massé shot. Because the friction force is nearly constant, the acceleration will also be nearly constant. This is the same situation as with projectiles in a gravitational field, which also experience constant acceleration. Hence, the CB curves with the same parabolic shape as the flight path of a projectile (e.g., a baseball or golf ball struck with no spin). The detailed math and physics can be found in **TP A.4**.
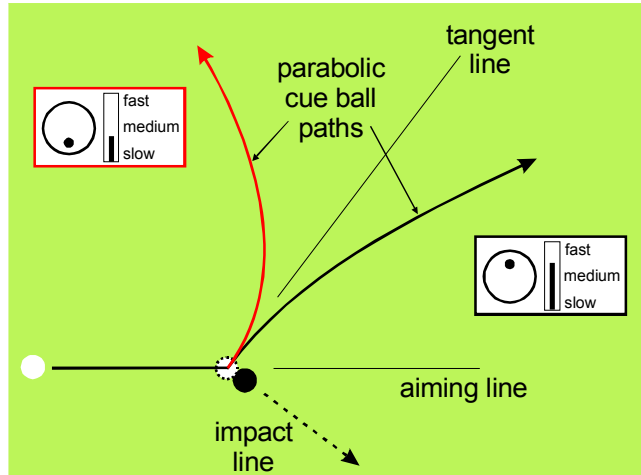
Figure 4 Parabolic CB paths

TP A.4 – Post-impact CB trajectory for any cut angle, speed, and spin

NV 4.20 – Delay of follow and tangent-line deviation with higher speed
NV 4.21 – Delay of draw and tangent-line deviation with higher speed

2. *To achieve maximum sidespin, the point of contact of the cue tip with the CB should be half a ball radius off center* (see **Figure 5**). Obviously, the farther you hit the CB off center, the more sidespin you impart. Although, if the tip offset exceeds the half-ball-radius (0.5R) amount, a *miscue* (where the cue tip slides off the CB during impact) is very likely (see **NV 2.1** and **HSV 2.1**). Needless to say, a miscue is undesirable in a game situation, especially if there is money on the line. The detailed math and physics behind miscues can be found in **TP 2.1**.
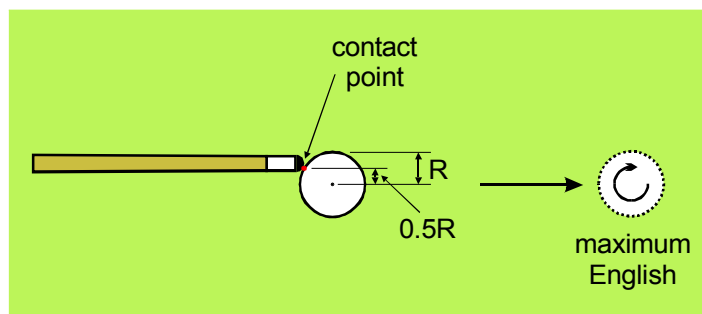

Figure 5 Contact point offset for maximum sidespin

NV 2.1 – Miscue due to off-center hit with no chalk

HSV 2.1 – Miscue due to off-center hit with no chalk

high-speed video

TP 2.1 – Minimum cue tip friction required for no-slip horizontal impact

technical proof

3. *With a massé shot, the final path of the CB will be in a direction parallel to the line drawn between the initial base point of the CB and the aiming point on the table.* The technique is illustrated in **Figure 6**. The final direction of the CB path is parallel to line *RA*, which connects the original CB resting point (point *R*) to the aiming point on the cloth (point *A*). Using the letters shown in the diagram, with "*B*" indicating the CB contact point, I refer to the Coriolis massé aiming system as the "*BAR*" method ("*B*" for ball, "*A*" for aim, and "*R*" for resting point). This technique can be useful when trying to aim massé shots, where you need to curve the CB around an obstacle ball (e.g., see **NV 7.11**). The detailed math and physics behind the BAR massé aiming method and resulting ball paths can be found in **TP A.19**.
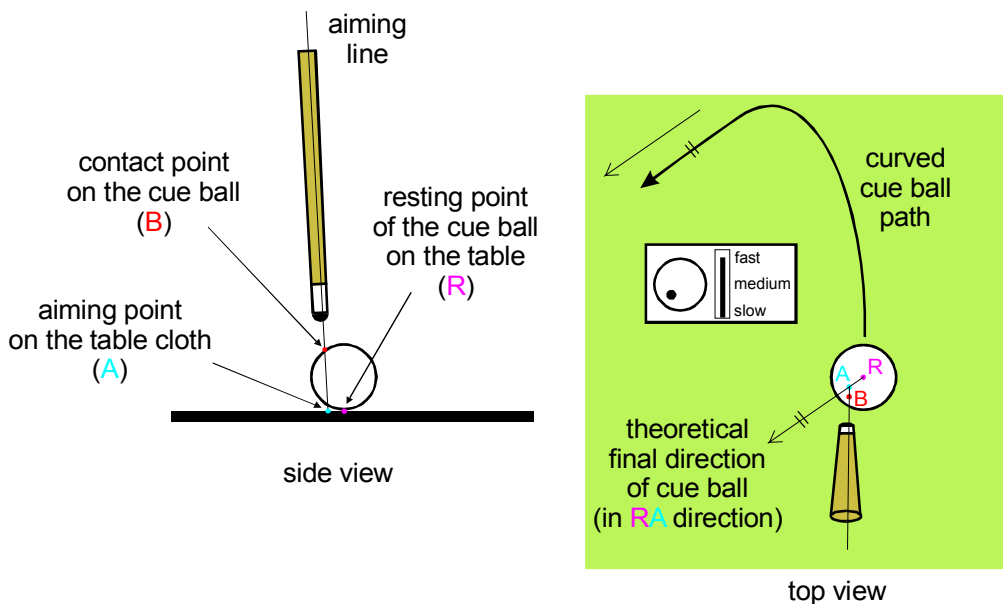

Figure 6 "BAR" massé shot aiming method

NV 7.11 – Large-curve massé shot

normal video

TP A.19 – Massé shot aiming method, and curved cue ball paths

technical proof

4. *For a CB with natural roll, the largest deflection angle the CB can experience after impact with an OB is 33.7°, which occurs at a cut angle of 28.1°* (see the next section for illustrations and a lot more info on why this is useful to know).

90º and 30º rules

Before looking closer at Coriolis' fourth conclusion in the previous section, I need to cover a little more basic pool physics. A very important skill for aiming and planning shots is the ability to predict where the CB will go after impact with an OB. The most important thing in pool is making a shot. The second most important thing is knowing where the CB will go so you can easily make the next shot. **Figure 7** illustrates one of the most important principles of pool related to this – the *90° rule* (see **NV 3.4**, **NV 3.5**, and **TP 3.1**). It states that when the CB strikes an OB with no topspin or bottom spin, the two balls will always separate at 90°. In other words, the CB will head exactly in the tangent line direction and persist along this line. This is true regardless of the cut angle (see **Figure 7**).
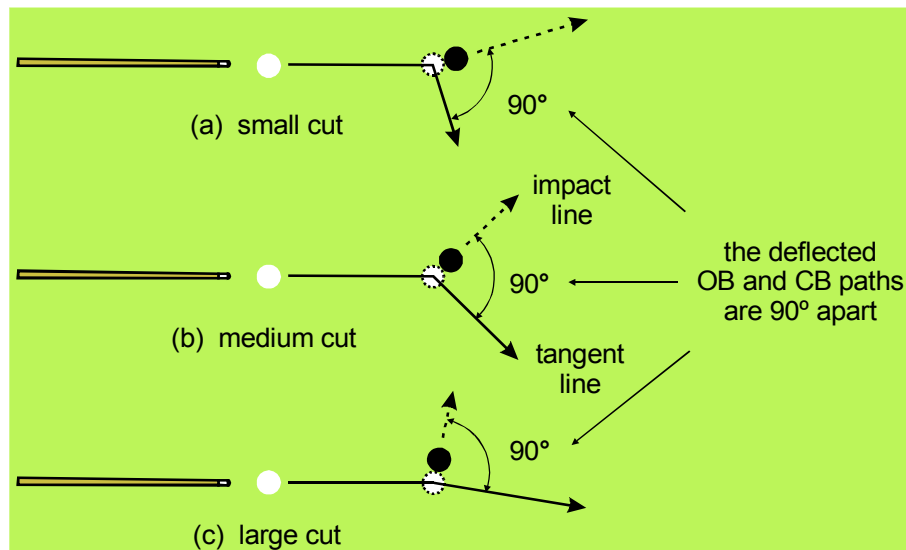

Figure 7 The 90° rule

NV 3.4 – 90° rule with various cut angles
NV 3.5 – Using your hand to visualize the 90° rule impact and tangent lines
NV 3.6 – Stop shot
NV 3.7 – Using the 90° rule to check for and prevent a scratch

TP 3.1 – 90° rule

The 90° rule is a direct result of the principles of conservation of energy and conservation of linear momentum (see the math and physics details in **TP 3.1**). All of the CB's momentum in the impact line direction (see **Figure 7b**) gets transferred to the OB. The result is that the final

paths of the balls are perpendicular. This is what happens ideally. In real life, there is a small amount of energy loss and retention of impact-line momentum when the CB hits the OB. This effect is quantified with what is called the *coefficient of restitution (COR)*. The COR for perfect (ideal) balls would be 1, representing 100% conservation of energy and complete momentum transfer. Typical pool balls have a COR closer to 0.93. The result is that the 90° rule is actually closer to the 85° rule (see **TP A.5**).

TP A.5 – The effects of ball inelasticity and friction on the 90° rule

technical proof

The 90° rule applies exactly only for a *stun shot*, where the CB is sliding without topspin or bottom spin at impact with the OB. The one exception to the 90° rule is when the CB hits the OB perfectly squarely, with no cut angle. In this case, the CB stops completely (i.e., it gets stunned into place), transferring all of its speed to the OB. This is called a **stop shot** (see **NV 3.6**, **HSV 3.1**, and **HSV 3.2**). Notice how in **HSV 3.1**, to achieve a stop shot, one must often hit the CB below center to create bottom spin, which wears off, so the CB has no spin at impact with the OB.

NV 3.6 – Stop shot

normal video

HSV 3.1 – Stop shot showing loss of bottom spin over distance
HSV 3.2 – Stop-shot to prevent a scratch

high-speed video

Figure 8 illustrates the physics involved with a stop shot. As the CB slides along the cloth, the friction force between the ball and cloth creates a torque about the ball's center that gradually reduces (decelerates) the bottom spin (see **Figure 8b**). If the CB were to continue to slide, the friction force would continue to change the CB's spin, slowly building up forward roll (e.g., top spin), as shown in **Figure 8a**. Note how the CB actually slows down (as indicated by the lengths of the straight arrows in the figure) as the topspin builds, because the sliding friction force is in the opposite direction as the ball's motion. **Figure 8c** shows what happens if the CB starts out with over-spin (more topspin than the roll amount). The excess quickly wears off, and the CB starts rolling. In this case, the CB actually speeds up a little because the sliding friction force actually pushes the ball forward. When the CB is rolling, there is no longer any sliding friction, and the CB continues to roll naturally until the ball slows to a stop or hits something (e.g., another ball or a rail). If the CB is struck with the cue tip at exactly the right height above center, the ball will roll immediately. The technical term for this height is the *center of percussion (COP)*, also known as the *sweet spot*, a term also used to describe the ideal impact location for a baseball bat or tennis racket. For a pool ball, the COP is at 70% of the ball's height above the table surface (see the math and physics details in **TP 4.2**). To get over-spin (e.g., as in **Figure 8c**), the CB must be struck above this height, which is risky due to likelihood for miscue.
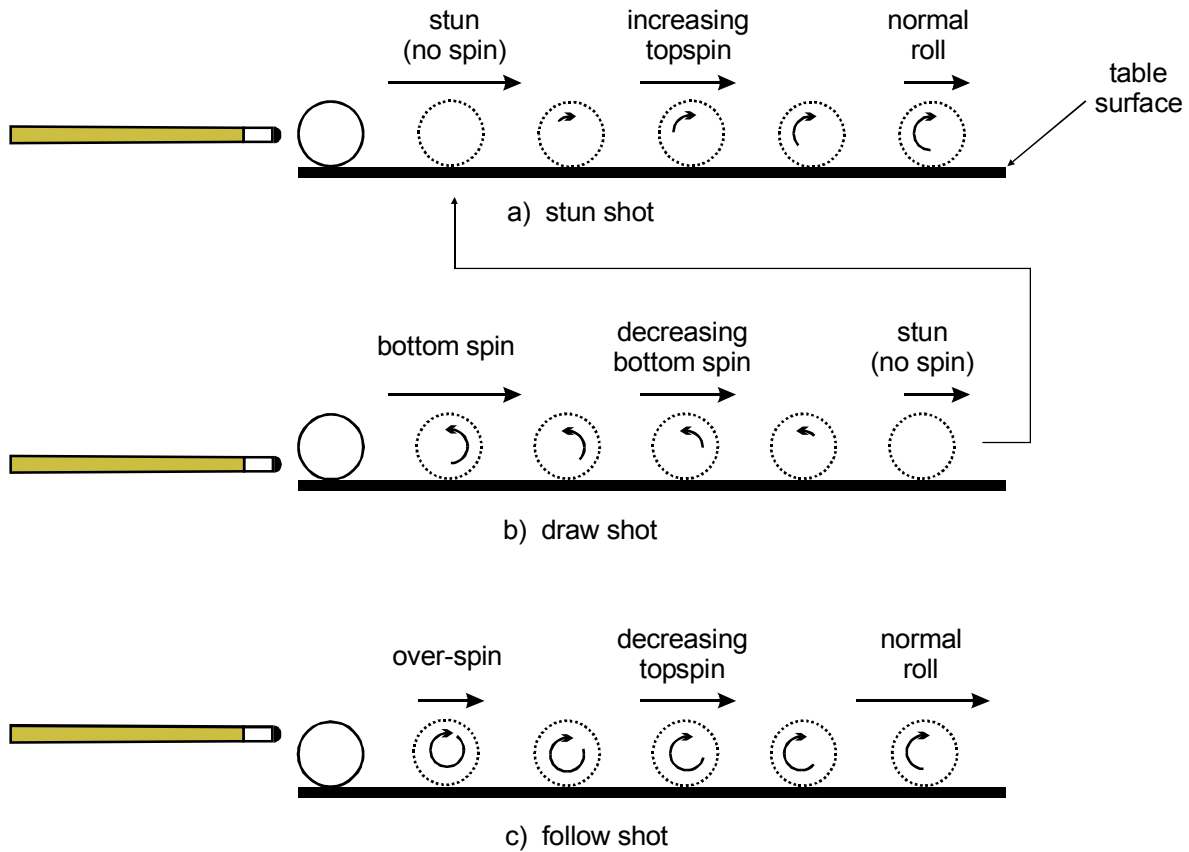
stun
(no spin)

increasing
topspin

normal
roll

table
surface

a) stun shot

bottom spin

decreasing
bottom spin

stun
(no spin)

b) draw shot

over-spin

decreasing
topspin

normal
roll

c) follow shot

Figure 8 Conversion of vertical-plane spin to normal roll (side view)

technical proof

TP 4.2 – Center of percussion of the cue ball

As described above, with most pool shots, the CB will be rolling by the time it reaches the OB. The exception is where bottom spin and/or fast speeds are being used. When the CB is rolling, the 90° rule no longer applies. What applies instead is governed by what I call the *30° rule* (see **Figure 9**, **NV 3.8**, **NV 3.9**, and **NV 3.10**). It states that when a rolling CB hits an OB close to a half-ball hit (see **Figure 10**), the CB will deflect approximately 30° away from its initial aiming line. In **TP 3.3**, I show the detailed physics and math behind the rule. I also show a modern proof for the numbers in Coriolis' 4th conclusion (see the previous section).
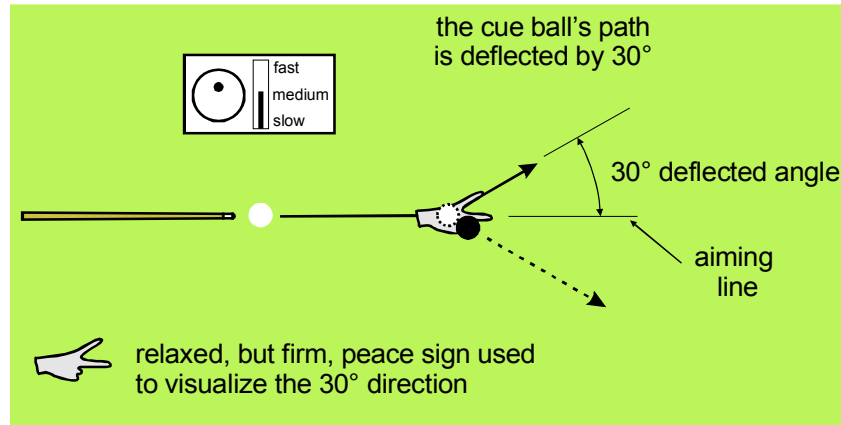
the cue ball's path
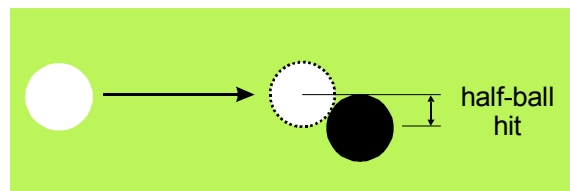is deflected by 30°

fast
medium
slow

30° deflected angle

aiming
line

relaxed, but firm, peace sign used
to visualize the 30° direction

Figure 9 30° rule

half-ball
hit

Figure 10 Half-ball hit

normal video

NV 3.8 – Using your hand to visualize the 30° rule
NV 3.9 – 30° rule example
NV 3.10 – Using the 30° rule to check for and prevent a scratch

technical proof

TP 3.3 – 30° rule

As shown in **Figure 9** and demonstrated in **NV 3.8**, you can use your hand to help visualize the 30° CB direction. As shown in **Figure 11**, if you form a relaxed but firm V-shape (peace sign or victory symbol) with your index and middle fingers, the angle between your fingers will be very close to 30°. **NV 3.8**, and **NV 3.9** show how to use the *Dr. Dave peace sign* in practice. If you point one of the fingers in the aiming line direction, the other finger will indicate the direction the CB will travel after impact.

You might be asking yourself: "How often does a pool player face half-ball hits, and is the 30° rule so useful after all?" Fortunately, as shown in **Figure 12**, the 30° rule applies over a wide range of ball-hit fractions (see **TP 3.3** for the math and physics details). The center of the range is the half-ball hit, but the CB deflection is very close to 30° for ball-hit fractions as small as 1/4 and as large as 3/4. **Figure 13** illustrates the ball-hit-fraction range, and corresponding cut angles, to illustrate the wide range of shots for which the 30° rule applies. With a 1/4-ball hit (see **Figure 13a**), the center of the CB is aimed outside of the object-ball edge such that the projected cue-ball-path passes through 1/4 of the OB. With a 1/2-ball hit (see **Figure 13b**), the center of the CB is aimed directly at the edge of the OB such that the projected cue-ball-path

passes through 1/2 of the OB. With a 3/4-ball hit (see **Figure 13c**), the center of the CB is aimed inside of the object-ball edge such that the projected cue-ball-path passes through 3/4 of the OB. These three cases cover a fairly large range of cut angles.
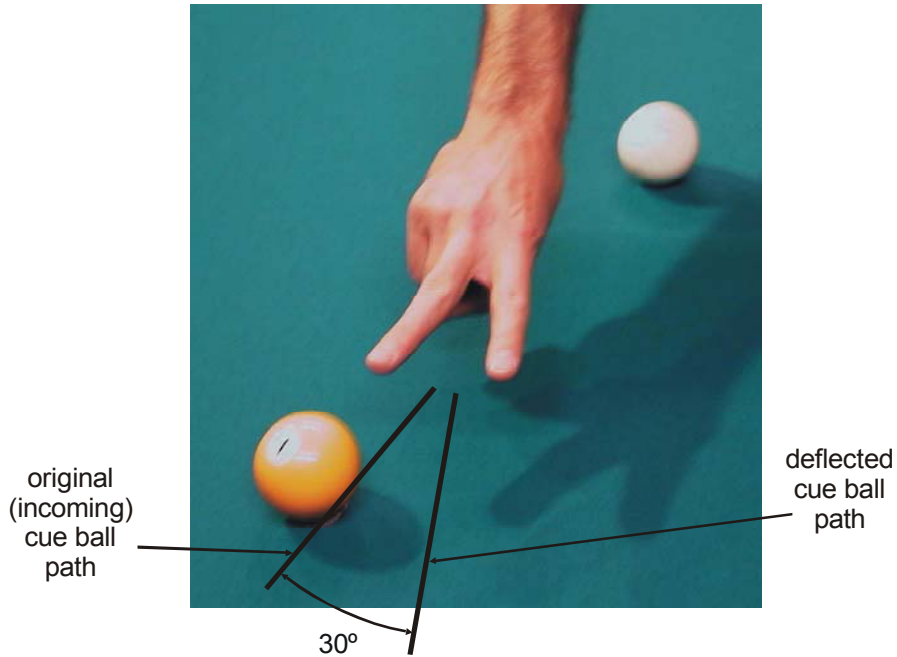

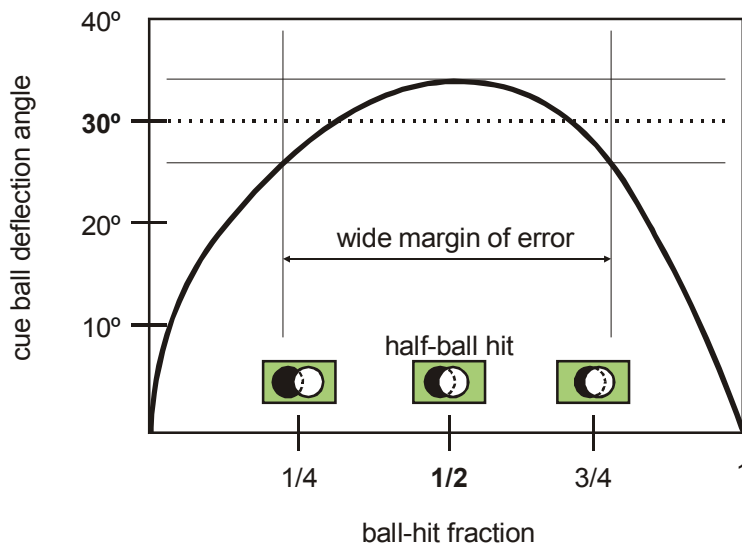**Figure 11 Using your hand to visualize the 30° rule CB paths**

Figure 12 Large margin of error for 30° rule

the cue ball path is deflected by approximately 30° in each case

top view:

large cut angle   ~30°   1/4

medium cut angle   ~30°   1/2

small cut angle   ~30°   3/4

side view:
(looking along the aiming line)

1/4   1/2   3/4

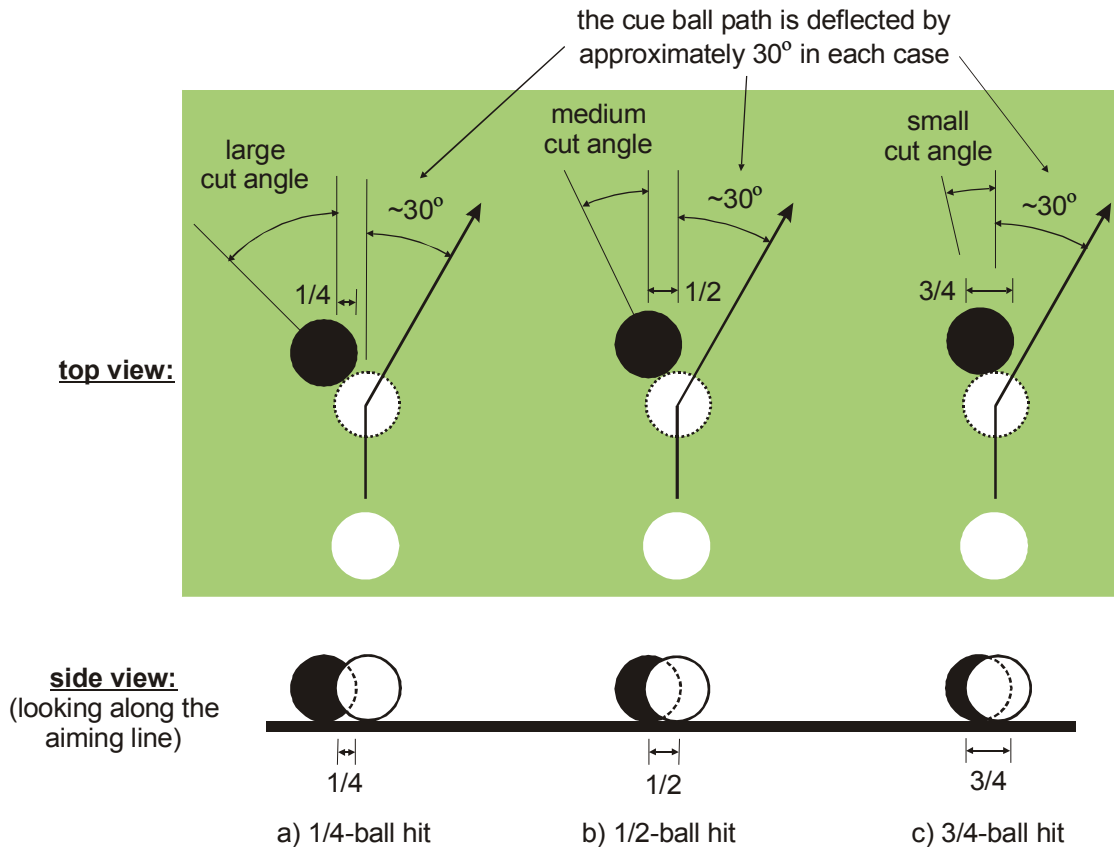a) 1/4-ball hit   b) 1/2-ball hit   c) 3/4-ball hit

Figure 13 Various ball-hit fractions

The 30° rule and peace sign technique are extremely useful in practice. This is a great example where a little knowledge of the physics of the game can be a big help. The rule can be used to detect possible scratches (where you pocket the CB by mistake), plan carom shots (where you deflect one ball off another into a pocket), plan ball avoidance or break-up shots, and strategically plan CB position as you run a rack of balls. Examples of all of these types of shots can be found on my website (e.g., see **NV 3.7**, **NV 3.10**, **NV 7.2**, **NV 7.3**, **NV 7.4**, and **NV A.1**).

NV 3.7 – Using the 90° rule to check for and prevent a scratch
NV 3.10 – Using the 30° rule to check for and prevent a scratch
NV 7.2 – Cue ball billiard shot
NV 7.3 – Object ball carom shot
NV 7.4 – 30° rule billiard shot
NV A.1 – Cut shot vs. carom shot example

normal video

Throw

An area of study I've spent much time on recently is the physics and applications of ball throw. *Throw* refers to the change in OB direction due to sliding friction forces between the CB and OB during impact. Because of throw, the OB doesn't go in the direction you might think it should (see **Figure 2**). **NV 4.15, 4.16, 7.5, and 7.6** show various examples of both **collision-induced throw**, caused by cut angle, and **spin-induced throw**, caused by sidespin.

Figure 14 illustrates all of the important terminology and physics concerning throw. Throw occurs any time there is relative sliding motion between the CB and OB surfaces at impact. In the figure, the relative motion is a result of CB sidespin, so the resulting throw would be called spin-induced throw. The right (counterclockwise) sidespin creates a sliding friction force that pushes the OB to the left. This force is what creates the *throw angle*. Because the *throwing force* is pushing on the edge of the OB, it also causes the OB to rotate about its center. Imagine pushing on the outside of a wheel ... the wheel will turn as a result. The spin imparted to the OB is called *transferred spin*. In this case, the throwing force to the left creates clockwise transferred spin on the OB. This can be important in certain types of shots (e.g., see **NV A.21**).
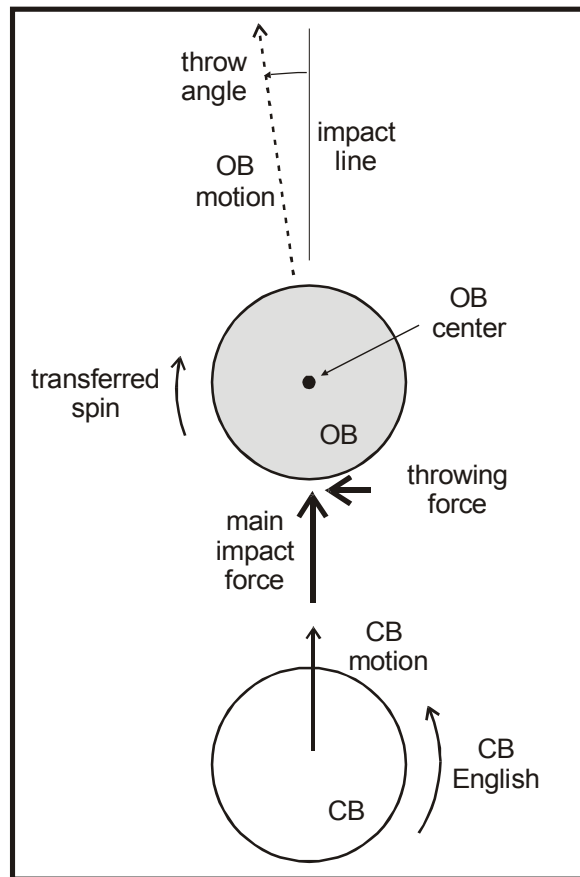

Figure 14 **Throw and spin transfer terminology and physics**

I am so excited about the math and physics of throw and spin transfer, I must share at least a small part of my throw analysis results with you. How about:

$$\theta_{throw} = \tan^{-1}\left\{\left[\min\left(\frac{\left(a_\mu + b_\mu e^{-c_\mu \sqrt{(v\sin(\phi)-R\omega_z)^2 + (R\omega_x \cos(\phi))^2}}\right)v\cos(\phi)}{\sqrt{(v\sin(\phi)-R\omega_z)^2 + (R\omega_x \cos(\phi))^2}}, \quad 1/7\right)(v\sin(\phi)-R\omega_z)\right]\Bigg/ v\cos(\phi)\right\}$$

Isn't it beautiful? Actually, I know that this *throw equation* probably isn't very pretty and just "looks like Greek" to most of you (as it should, because it has many Greek letters in it), but please bear with me. Amazingly, this single equation explains practically every effect and principle related to throw, both collision-induced and spin-induced. The equation provides insight into the effects of speed, cut angle, sidespin, and ball friction, and the results are in agreement with numerous observations and experiments. The complete math and physics details can be found in **TP A.14**.

TP A.14 – The effects of cut angle, speed, and spin on OB throw

technical proof

Figure 15 shows an example plot generated with the throw equation. This particular plot shows how the amount of throw changes with cut angle for a stun shot (i.e., sliding CB) at various speeds (slow, medium, and fast). Useful conclusions one can observe from the figure include:
- for small cut angle shots (i.e., fuller hits), the amount of throw does not vary with shot speed, but increases with cut angle.
- for larger cut angle shots (i.e., thinner hits), the amount of throw is significantly larger for slower speed shots as compared to faster speed shots.
- the amount of throw decreases a little at large cut angles, but not by much (especially for slower speed shots).
- the maximum throw occurs at close to a half-ball hit (30º cut angle).

All of this information is useful to know at the table; and if you don't know how to adjust for these effects (consciously or subconsciously), you won't be a great player.
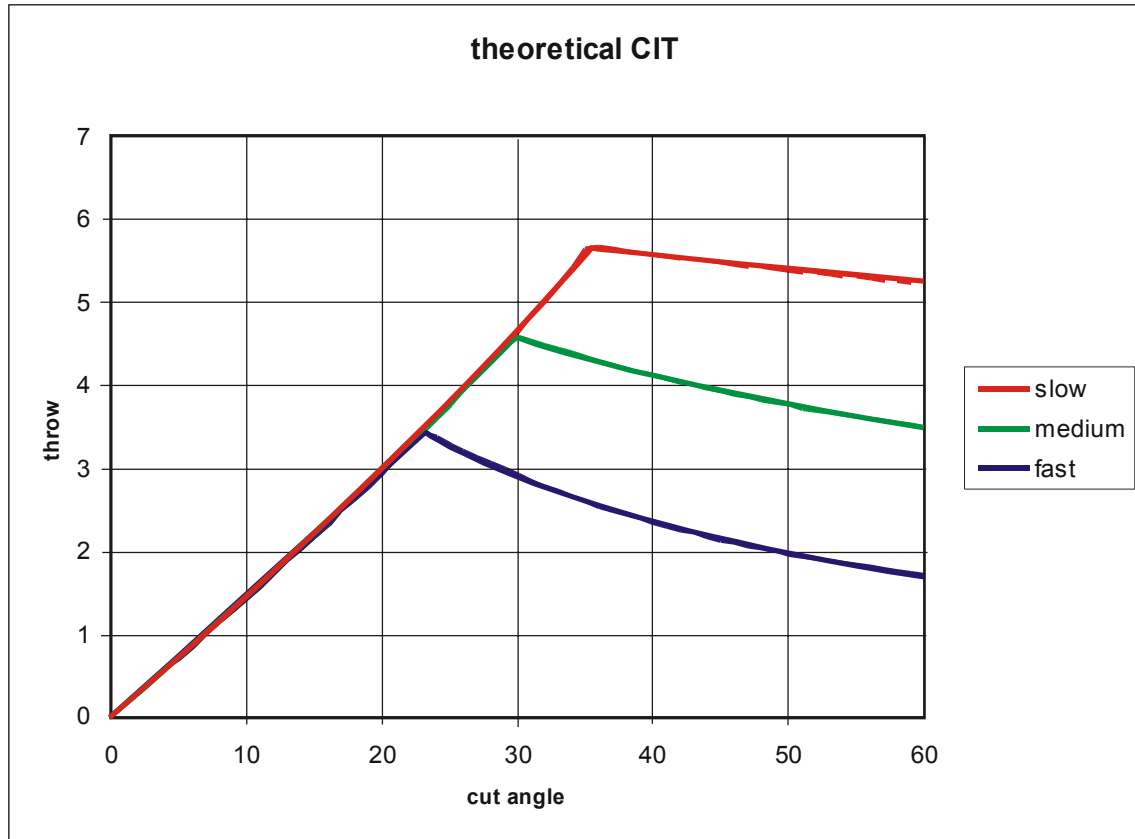
theoretical CIT

Figure 15 Stun shot collision-induced throw

Many more plots like **Figure 15**, which apply to various types of shots, can be found in **TP A.14**. Also, many more conclusions and results are summarized that can be useful to know in game situations (see the instructional articles on my website for more information).

Conclusions

Well, I hope you have enjoyed my article. I also hope that you now agree that a little knowledge of pool physics might even be useful (or at least, interesting). If you're still not convinced, check out more of my instructional articles and video demonstrations online at **billiards.colostate.edu**. Also, if you like the super-slow-motion high-speed-video stuff, check out **high speed video.colostate.edu**. There, you can find almost everything imaginable including shooting a shaken pop can with a pellet gun, dropping a cube of Jell-O, bouncing and puncturing water balloons, and dropping an egg on a rat trap. You can even see performances of various stupid human and animal tricks ... all in super-slow-motion for your viewing pleasure. Enjoy!

If you want to learn more about playing and understanding the physics of pool, I've provided some useful references below. However, I should warn you that references 1, 2, and 3, and the TPs on my website, are quite technical and require strong math and physics backgrounds. If you care more about just playing pool better, I recommend references 4 and 5 and the instructional

articles I have posted online. Also on the website are many links to online articles, videos, and resources available from others. There is a wealth of pool information available on the Internet.

In closing, in the spirit of the 30° rule:

If you let one finger stay,
The other finger points the way.
Peace.

Good luck with your game,
Dr. Dave

Bibliography

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